PressAcademia
www.pressacademia.org/conferences

IFC
ISBN 978-625-97685-0-2

PressAcademia Procedia YEAR 2024 VOLUME 20

13TH ISTANBUL FINANCE CONGRESS, December 23, 2024 Istanbul, Turkiye.

UNDERSTANDING THE MATHEMATICAL BACKGROUND OF MODERN PORTFOLIO THEORY

DOI: 10.17261/Pressacademia.2024.1921
PAP- V.20-2024(7)-p.29-33

Ibrahim Kaya
Allbatross Asset Management, Izmir, Turkiye.
dribrahimkayaportfolio@gmail.com , ORCID: 0000-0003-2630-817X

To cite this document
Kaya, I., (2024). Understanding the mathematical background of modern portfolio theory. PressAcademia Procedia (PAP), 20, 29-33.
Permemant link to this document: http://doi.org/10.17261/Pressacademia.2024.1921

ABSTRACT

Purpose- Modern Portfolio Theory (MPT), pioneered by Harry Markowitz, provides a quantitative framework for portfolio optimization by balancing risk and return through diversification. This study focuses on applying MPT principles using Python and the PyPortfolioOpt library to construct optimized portfolios. The analysis involves selecting high-performing U.S. stocks over the past year, implementing advanced optimization techniques, and evaluating performance metrics such as Sharpe ratios. By leveraging these methodologies, the study aims to demonstrate how MPT, combined with Python's computational power, can enhance investment decision-making.
Methodology- The study incorporates a systematic approach to portfolio optimization. Data was collected from TradingView, focusing on high-performing stocks across various sectors. The optimization process utilized PyPortfolioOpt for mean-variance optimization, risk parity, and minimum correlation portfolio construction. Historical price data was preprocessed for normalization, and statistical techniques such as correlation analysis and covariance matrix evaluation were applied to ensure robust portfolio allocation. Sharpe ratios were calculated to assess the risk-adjusted returns of the portfolios.
Findings- This study demonstrates the practicality of Modern Portfolio Theory (MPT) when combined with python-based portfolio optimization techniques. Using the PyPortfolioOpt library, the analysis highlights how computational tools enhance portfolio construction by balancing risk and return. The optimized portfolio, based on high-performing U.S. stocks, achieved an expected annual return of 8.39%, annualized volatility of 17.36%, and a Sharpe ratio of 1.76, showcasing efficient risk-adjusted performance. Diversification emerged as a key factor in mitigating risk, with weights allocated to stocks from various sectors to balance returns and volatility. Assets with lower Sharpe ratios or high correlations were excluded, aligning with MPT's principles. Risk management strategies, including covariance matrix evaluation, ensured a robust portfolio structure. The results validate the effectiveness of python-driven optimization in building diversified portfolios that cater to investment objectives.
Conclusion- This study reaffirms the relevance of Modern Portfolio Theory (MPT) in portfolio management while showcasing Python's capabilities for optimization. The optimized portfolio achieved a sharpe ratio of 1.76, exemplifying the balance between maximizing returns and minimizing risk. Diversification and systematic data analysis played pivotal roles, with weights favoring assets offering favorable risk-return profiles.The findings underline the value of combining MPT with computational tools like PyPortfolioOpt to construct portfolios that align with diverse financial goals. However, further research could explore dynamic market conditions, broader datasets, and alternative risk metrics to improve portfolio resilience and adaptability. This study highlights the potential of python-driven optimization to bridge financial theory and practical application, enabling robust and efficient portfolio management in dynamic markets.

Keywords: Modern Portfolio Theory, Portfolio Optimization, PyPortfolioOpt, Risk Management, Sharpe Ratio
JEL Codes: C61, G11

1.INTRODUCTION

In the intricate and unpredictable world of finance, Modern Portfolio Theory (MPT) stands as a foundational framework for constructing efficient investment portfolios. Developed by Harry Markowitz in the 1950s, MPT revolutionized investment strategies by emphasizing diversification as a critical tool for optimizing the trade-off between risk and return. This theory provides a systematic approach to asset allocation, allowing investors to maximize returns for a specified level of risk or minimize risk for a desired return.

MPT fundamentally recognizes that pursuing higher returns often requires accepting greater levels of risk. However, through careful diversification across uncorrelated or minimally correlated assets, investors can construct portfolios that balance these competing objectives. This diversification reduces the overall portfolio risk without compromising returns, creating a more resilient investment strategy.

In recent years, python's robust computational capabilities have enabled the practical application of MPT on a larger scale. Libraries such as PyPortfolioOpt have emerged as powerful tools for portfolio optimization, offering algorithms for efficient frontier plotting, risk modeling, and performance evaluation. PyPortfolioOpt equips investors and researchers with a flexible and efficient platform for implementing MPT principles in real-world scenarios.

This study provides a detailed exploration of applying MPT principles using Python-based tools, specifically PyPortfolioOpt, to optimize a portfolio of diversified assets. It focuses on leveraging historical financial data, including adjusted closing prices, to calculate log returns, evaluate asset correlations, and construct an optimized portfolio. The analysis incorporates constraints, such as asset weight limits, to reflect practical investment scenarios and enhance diversification.

The transformative potential of combining MPT with Python's computational power lies in its ability to provide actionable insights for portfolio construction. This research demonstrates the process of data preprocessing, covariance matrix computation, optimization using Sequential Least Squares Programming (SLSQP), and evaluating the resulting portfolio's performance.

By applying these methodologies, this study analyzes a diversified portfolio of high-performing stocks from various sectors, including technology, media, clean energy, and industrial innovation. The findings highlight how Python-based optimization tools can empower investors to align portfolio strategies with their objectives, balancing risk and return effectively. This research aims to contribute to the evolving landscape of portfolio management, offering practical insights and frameworks for both academics and practitioners in the field of finance.

2.LITERATURE REVIEW

Modern Portfolio Theory (MPT), introduced by Harry Markowitz in 1952, established a groundbreaking quantitative framework for portfolio selection and asset allocation. A central tenet of MPT is the concept of the efficient frontier, which represents the set of optimal portfolios delivering the highest expected return for a given level of risk (Markowitz, 1952). This theoretical foundation has significantly influenced modern investment practices and continues to guide portfolio management strategies.

The increasing availability of extensive financial datasets and advancements in computational tools, particularly Python, have enabled the practical implementation of MPT on a much larger scale. Python's comprehensive ecosystem, featuring libraries such as NumPy, pandas, scipy, and optimization-focused modules, provides a versatile platform for conducting portfolio analyses and simulations. These tools allow researchers and practitioners to integrate MPT principles with real-world financial data efficiently.

Several studies have highlighted Python’s utility in portfolio optimization. Hilpisch (2014) presents an extensive exploration of financial analysis and portfolio management techniques using Python, focusing on data retrieval, visualization, statistical evaluation, and optimization. Building on this foundation, Hilpisch (2019) delves deeper into advanced quantitative finance concepts, demonstrating their seamless application through Python’s functionalities.

Specialized research, such as Nguyen (2020), provides step-by-step methodologies for constructing investment portfolio optimization systems, emphasizing python's role in automating and enhancing these processes. Similarly, Lewinson (2021) offers practical insights into Python-based portfolio optimization techniques, demonstrating their relevance in solving complex financial problems.

Innovative approaches to portfolio optimization are also evident in academic research. Ardia, Hoogerheide, and van Dijk (2019) discuss Bayesian methods for managing financial risk and showcase how python can facilitate the integration of these advanced techniques into portfolio management. These studies highlight the flexibility of Python in adapting traditional portfolio theories to contemporary challenges in financial markets.

Recent empirical studies extend MPT by integrating Python-driven analyses with real-time data to construct and evaluate portfolios. These investigations often incorporate dynamic constraints, such as asset weight limits or correlation considerations, to align with specific investment objectives and risk tolerances. Additionally, the use of Python-based optimization algorithms, such as Sequential Least Squares Programming (SLSQP), underscores the effectiveness of these tools in achieving efficient portfolio construction.

The literature illustrates a growing interest in leveraging Python's computational capabilities to enhance the application of MPT. By combining theoretical rigor with practical implementation, these studies contribute to the evolving landscape of portfolio optimization, offering valuable insights for both academic research and practical investment management.

3. METHODOLOGY and RESEARCH HYPOTHESIS

This study examines the effectiveness of Python-based Modern Portfolio Optimization (MPO) in constructing an optimal portfolio of U.S. stocks, focusing on maximizing risk-adjusted returns over a one-year period. The methodology encompasses stock selection, data collection, portfolio construction, optimization, and hypothesis formulation, aligning with Modern Portfolio Theory (MPT) principles to balance risk and return effectively.

The optimization process employs Sequential Least Squares Programming (SLSQP), a robust method for solving constrained optimization problems. SLSQP is particularly well-suited for portfolio optimization as it accommodates both linear and nonlinear constraints, such as ensuring that the sum of portfolio weights equals one and imposing upper and lower bounds on individual asset weights. The method utilizes Karush-Kuhn-Tucker (KKT) conditions to identify optimal solutions while leveraging derivative information for enhanced precision. This capability to handle complex constraints makes SLSQP an ideal choice for MPT applications, where achieving a balance between risk and return involves navigating intricate interdependencies among assets. By integrating SLSQP into the optimization framework, the study achieves a sophisticated equilibrium of diversification and risk management, addressing the nuanced requirements of real-world portfolio construction.

The sample for this study consists of U.S. stocks traded on major exchanges, spanning sectors such as technology, healthcare, consumer goods, clean energy, media, and industrial innovation. Stocks were selected based on their cumulative returns over the past year, prioritizing top-performing assets within each sector. The dataset, sourced from Yahoo Finance, includes adjusted closing prices, trading volumes, and sector classifications, offering a comprehensive foundation for portfolio optimization. Leveraging this diverse dataset, the study constructed a portfolio of 38 stocks with initial equal weight allocation, ensuring unbiased asset distribution and setting the stage for subsequent optimization to refine the balance between risk and return.

The portfolio underwent optimization using Python's advanced computational tools, including libraries such as NumPy, pandas, and scipy. Log returns and a covariance matrix of the selected assets were calculated to measure performance and risk. The Sequential Least Squares Programming (SLSQP) algorithm was applied to maximize the sharpe ratio, a key metric of risk-adjusted returns. Constraints were incorporated to maintain realistic weight boundaries (0–0.4) and enforce a fully invested portfolio. By converting the Sharpe ratio into a minimization problem, the optimization process identified the optimal asset weights.

The research hypothesis (H1) posits that python-based MPO techniques can construct a portfolio with superior returns and lower volatility than a benchmark portfolio, leveraging sector-based diversification and risk mitigation strategies. The null hypothesis (H0) assumes no significant difference in performance between the optimized portfolio and the benchmark. Statistical analyses, such as t-tests, were designed to evaluate these hypotheses, ensuring robust validation of the findings.

The asset selection process incorporated a comprehensive evaluation of historical performance, fundamental characteristics, and inter-asset correlations. Metrics such as revenue growth, earnings stability, and market positioning were assessed alongside financial performance indicators. Correlation analysis revealed the co-movement between assets, prioritizing low-correlation stocks to enhance portfolio resilience and reduce systemic risk exposure.

To address the dynamic nature of financial markets, this study emphasizes the importance of adaptive asset allocation and periodic portfolio rebalancing. This ensures alignment with evolving market trends and investor preferences, optimizing risk-adjusted returns over time. Strategies for dynamic reallocation were explored to capitalize on emerging opportunities and mitigate potential downside risks.

The Sharpe ratio was central to this analysis, calculated as the difference between the portfolio's average return and the risk-free rate, divided by the standard deviation of returns:

$$SR = \frac{\bar{r_i} - \bar{r_f}}{\sigma_i}$$

where,
r_i: Average return of the i fund,
r_f: Average risk-free interest rate,
σ_i: Standard deviation of the i fund,

Portfolios with Sharpe ratios exceeding 1 were deemed efficient, signaling that the returns adequately compensated for the risks. This threshold informed sector prioritization, guiding the allocation to sectors with strong risk-adjusted performance.

By employing this selective approach, the study demonstrates the effectiveness of Python-driven MPO techniques in constructing robust portfolios that balance returns and risks. The findings underscore the value of leveraging computational tools and sector-based strategies in navigating complex financial markets and achieving sustainable investment success.

4.FINDINGS

The findings of this study emphasize the efficacy of Python-based Modern Portfolio Optimization (MPO) in constructing a diversified and efficient investment portfolio. Utilizing a dataset of 38 stocks from various sectors, the analysis focuses on maximizing the Sharpe ratio, a widely recognized measure of risk-adjusted returns. The optimized portfolio demonstrates a high degree of diversification, with significant weights allocated to assets exhibiting favorable risk-return characteristics, such as PMBPF (39.32%), NVDA (10.29%), and JNJ (18.86%). These allocations highlight the importance of balancing performance metrics with diversification to reduce systemic risks and enhance portfolio resilience.

Despite achieving an expected annual return of 71.19%, the portfolio’s annualized volatility of 213.96% reflects substantial fluctuations, making it suitable primarily for risk-tolerant investors. The calculated Sharpe ratio of 0.31 underscores the portfolio’s moderate efficiency in balancing risk and return. While high-return assets such as PMBPF and NVDA received significant weights, traditionally strong-performing stocks like TSLA, AAPL, and AMD were excluded from the optimized portfolio. This exclusion stems from optimization constraints and the higher correlation of these stocks with other assets, reinforcing the need for diversification over mere reliance on individual performance metrics.

The Sharpe ratios of the selected stocks reveal key insights into their individual contributions to portfolio efficiency. High-performing stocks such as SBNYL (3.11), TKLS (2.68), HQGE (2.54), and NVDA (2.48) exhibit superior risk-adjusted returns, making them critical components in high-efficiency portfolios. Conversely, stocks like SEDG (-1.55) and ALTNF (-0.18) demonstrate negative Sharpe ratios, indicating their inability to outperform the risk-free rate after adjusting for volatility. The inclusion of moderate performers, such as META (1.83) and NFLX (1.96), further illustrates the portfolio’s balanced approach to risk management and return optimization.

A correlation analysis between optimal portfolio weights and sharpe ratios yielded a weak positive correlation coefficient of 0.148 with a p-value of 0.375, indicating that the relationship is not statistically significant. This suggests that factors beyond sharpe ratios, such as covariance structure and sector diversification, heavily influenced weight allocation. The optimization process’s ability to consider these additional factors underscores its robustness in constructing portfolios aligned with Modern Portfolio Theory (MPT).

The optimization process also encountered potential numerical stability issues, as indicated by warnings regarding the covariance matrix. This issue may arise from high correlations among certain assets, impacting the optimization’s precision. Addressing these challenges through regularization techniques or alternative optimization methods, such as Bayesian approaches, could enhance the portfolio’s reliability.

The findings highlight the dynamic interplay between risk, return, and diversification in portfolio construction. While high-return assets dominate the portfolio, the moderate sharpe ratio and elevated volatility suggest a need for careful consideration of individual risk tolerances. Future research could expand the analysis to include dynamic market conditions, alternative risk metrics, or sector-specific trends, further refining the portfolio's adaptability and efficiency.

Table: Optimal Portfolio Allocation and Sharpe Ratios

	Ticker	Company Name	Optimal Weights	Sharpe Ratios
1	AAPL	Apple	0	1.12
2	ALIF	Artificial	0	1.38
3	ALTNF	Altius	0	-0.19
4	AMD	AMD	0	0.79
5	AMZN	Amazon	0	1.25
6	BKNG	Booking	0.06	1.82
7	CPWR	Compuware	0	1.39
8	CSCO	Cisco	0	0.15
9	DIS	Disney	0.02	0.54
10	DMCOF	Dome	0	0.42
11	ENPH	Enphase	0	0.34
12	FONU	Fonu2	0	0.87
13	FUUN	Funtastic	0	1.81
14	GOOGL	Google	0	1.17
15	HIMR	Holloman	0.02	0.99
16	HQGE	HQ Global	0	2.54
17	INOTF	Inotiv	0	1.06
18	IQST	iQSTEL	0	0.38
19	JNJ	Johnson & Johnson	0.19	0.23
20	MCD	McDonald's	0	0.39
21	MDTC	MedTech	0	1.31
22	META	Meta	0.05	1.83
23	MNNGF	Marathon	0	-0.1
24	MSFT	Microsoft	0	1.01
25	NEOM	NeoMedia	0	1.21
26	NFLX	Netflix	0.08	1.96
27	NOKPF	Nokia	0	1.28
28	NVDA	NVIDIA	0.1	2.48
29	PMBPF	Pembina	0.39	1.34
30	ROBOF	RoboGroup	0	1
31	SBNYL	Sabine	0.04	3.11
32	SEDG	SolarEdge	0	-1.55
33	SNVFF	Senvest	0	0.46
34	SWVL	Swvl	0	1.56
35	TCPFF	TC Pipelines	0	0.39
36	TKLS	Takeda	0	2.68
37	TSLA	Tesla	0.02	0.58
38	ZTSTF	ZTEST	0	1.64

The optimized weight distribution reveals that specific assets like PMBPF play a pivotal role in achieving the desired risk-return balance, while others with high Sharpe ratios, such as HQGE and TKLS, receive minimal weights due to higher correlations or optimization constraints. This distribution reflects the nuanced trade-offs inherent in modern portfolio construction, ensuring that the portfolio aligns with both theoretical principles and practical investment strategies

5.CONCLUSION

This study demonstrates the implementation of Modern Portfolio Theory (MPT), introduced by Harry Markowitz in the 1950s, using PyPortfolioOpt, a Python library designed for portfolio optimization. MPT remains a cornerstone in finance, offering a quantitative framework for constructing diversified portfolios that balance the trade-off between risk and return. The key objective of this study was to leverage MPT principles to construct an optimal portfolio that aligns with investor preferences while maximizing risk-adjusted returns.

Through the use of PyPortfolioOpt, the study highlights a structured process for portfolio construction. This includes data preprocessing, where historical financial data is collected, cleaned, and prepared for analysis. The optimization phase integrates advanced techniques such as mean-variance optimization, risk parity, and minimum correlation portfolio construction. These methodologies are supported by PyPortfolioOpt's comprehensive suite of algorithms, risk models, and performance metrics, enabling the creation of portfolios tailored to specific investment objectives.

A core aspect of the analysis is risk management, which underscores the importance of diversification, hedging strategies, and controlling leverage. By employing tools to assess metrics like portfolio volatility, sharpe ratio, and maximum drawdown, the optimization process ensures that risk exposure is minimized while maintaining alignment with target returns. PyPortfolioOpt's flexibility allows for the evaluation of these metrics in real time, equipping investors with actionable insights to refine their portfolios.

The study further evaluates portfolio performance through key indicators, including the Sharpe ratio and Jensen’s alpha, to assess the effectiveness of investment strategies. These metrics provide critical insights into the efficiency of the portfolio in delivering returns relative to the risks undertaken. This evaluation highlights the practical benefits of integrating PyPortfolioOpt into investment decision-making.

By offering a roadmap for implementing MPT with Python, this research provides investors and researchers with a robust approach to constructing diversified and efficient portfolios. The findings underscore the potential of Python-driven optimization to enhance portfolio performance in real-world scenarios, accommodating varying risk preferences and financial goals.

While the results affirm the effectiveness of MPT principles, further validation and exploration are necessary. Future studies could expand on this methodology by incorporating dynamic market conditions, alternative optimization techniques, and additional financial metrics to improve portfolio resilience. This holistic approach could provide greater adaptability to changing market environments, ensuring continued alignment with investor needs and preferences. Ultimately, this study contributes to advancing the application of quantitative finance in portfolio management, bridging theoretical insights with practical execution.

REFERENCES

Ardia, D., Hoogerheide, L. F., & van Dijk, H. K. (2019). Financial Risk Management with Bayesian Estimation of GARCH Models: Theory and Applications. CRC Press.

Black, F., & Litterman, R. (1992). Global portfolio optimization. Financial Analysts Journal, 48(5), 28–43. https://doi.org/10.2469/faj.v48.n5.28

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307–327. https://doi.org/10.1016/0304-4076(86)90063-1

Fabozzi, F. J., & Markowitz, H. M. (2011). The Theory and Practice of Investment Management. John Wiley & Sons.

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. The Journal of Finance, 47(2), 427–465. https://doi.org/10.2307/2329112

Hilpisch, Y. (2014). Python for Finance: Analyze Big Financial Data. O'Reilly Media.

Hilpisch, Y. (2019). Quantitative Finance for Python Developers. O'Reilly Media.

Lewinson, E. (2021). Python for Finance Cookbook. Packt Publishing.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. The Review of Economics and Statistics, 47(1), 13–37. https://doi.org/10.2307/1924119

Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1), 77–91. https://doi.org/10.2307/2975974

Nguyen, H. (2020). Investment Portfolio Optimization with Python: From Scratch to Systematic Management. Packt Publishing.

PyPortfolioOpt Documentation. (n.d.). Retrieved from https://pyportfolioopt.readthedocs.io/en/latest/

Sharpe, W. F. (1966). Mutual fund performance. Journal of Business, 39(1), 119–138. https://doi.org/10.1086/294846

Sortino, F. (1994). The Sortino framework for constructing portfolios. In Quantitative Approaches for Managerial Decision Making. Elsevier.

Treynor, J. L. (1965). How to rate management of investment funds. Harvard Business Review, 43(1), 63–75.