



## EXPENDITURE OF FIRMS ON R&D IN DIFFERENT STRUCTURAL MARKETS

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### ABSTRACT

The focus in this paper is on structural and cooperative reasons of a market contribute to maximizing the expenditure of firms on R&D. We conclude that decline intensity of competition and increase the density of cooperation are important matters in increasing the expenditure on R&D. This in turn has a role in enhancing the individual and social outcomes. Moreover, growth of the market size does not necessarily imply low investments. The results suggest that the competition intensity determines the investment rate acquired from increasing new firms.

**Keywords:** R&D expenditure; Market structure; Network size; Equilibrium outcome

**JEL Classification:** D21, D92, L11

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### 1. INTRODUCTION

Research and development (R&D) is defined as activities that are achieved by one agent or a group of agents to create a new discovery or knowledge that may be used for new applications. It is an essential source of innovation, which is in turn an important factor in the growth of output in the economy as a whole. For firms, participating in R&D maintains their positions in a market through reducing the cost of the production (cost-reducing alliances), enhancing or developing existing products, finding new processes or producing new technologies that could open up new markets (Hagedoorn, 1993; Hagedoorn et al., 2000; Hagedoorn, 2002; Mowery et al., 1998; Powell, 1998; Goyal and Moraga-Gonzalez, 2001; Brown and Petersen, 2009).

We consider the case when firms conduct R&D to reduce the cost of the production under a network game. The importance of introducing the network concept to R&D model appears through contributing tools from the former theme to understanding the effects of R&D agreements on expenditures by firms. In addition, the network approach exhibits several categories of R&D partnerships, so that different structures of collaborative networks can be described. From these networks there are individually and socially desirable networks that may consist with each other.

The R&D network model used in this paper is based on Goyal and Moraga-Gonzalez (2001), and it can be briefly described as follows. The structure that displays firms cooperate in R&D is described as an R&D network where the players (firms) are represented by nodes and the R&D partnerships (agreements) are represented by links. The model consists of three stages: network formation, R&D investment and market competition. The marginal cost of the production decreases with increasing the individual investment and investment of other firms in the network, depending on R&D spillovers. Specifically, if any two firms cooperate, they are linked in the network and the spillover between them is set one; otherwise there is a free spillover less than one (the ability to take advantage of partners' R&D investment).

In Goyal and Moraga-Gonzalez model, the cost of the cooperation is ignored and the spillover is set as a free parameter from the network structure. The authors considered Cournot competition for regular and irregular networks for independent and homogeneous goods. They found that if goods are independent, the equilibrium outcomes increase with growing R&D agreements. They also found that if goods are homogeneous, the individual profit increases with agreements, but the individual investment and social welfare are negatively affected by the R&D agreements.

The aim of this paper is to investigate theoretically the relationship between the financing of R&D by firms and the formation of the market and the network. In particular, we use a linear inverse demand function under Cournot competition for independent and homogeneous goods to answer the following questions:

- 1) What are the factors that affect the finance of R&D?
- 2) What are the impacts of the investment and cooperation of firms in R&D on the individual and social outcomes?
- 3) How does increase of the number of firms in a marketplace affect the financing of R&D?

The outcomes of this paper can be summarized as follows. Firstly, we study some factors affecting the investment of firms in R&D. The first factor is the market structure and this means the impact of possible relationships between products on investment in R&D. There is a reverse relationship between differentiation rate of the products and the expenditure of firms on R&D i.e., the increase of the differentiation rate causes a decline in the expenditure. Meaning that, the investment of firms in R&D for differentiated goods is higher than the investment for substituted goods. The second factor is the network structure and this means the impact of the R&D cooperative relationships and the spillover on the investment. If firms are in a highly differentiated market, developing the network structure encourages firms to invest in R&D. Also, the R&D spillover resulting from building the network always enhances the investment. In contrast, if firms are in a highly substituted market, the R&D agreements and spillovers reduce the investment. This suggests that the expenditure of large firms on R&D is massive in a differentiated market, but low in a substituted market.

Secondly, we examine the role of the investment and cooperation in R&D on the outcomes. The results emphasize the importance of investment in R&D, but the benefit behind the cooperation depends on the market structure. In a differentiated market, the relative increase in spending on R&D and forming new partnerships always lead to a rise in the individual and social outcomes. Also, the increase of the R&D spillover positively affects the equilibrium outcomes. However, in a substituted market, the increase of the R&D agreements of the individuals leads to (i) a large decline in the social outcomes for most values of the spillover, (ii) rise in the individual profit. The positive relation between the payoff and the R&D agreements in all market cases reflects the rising importance of the cooperation to remaining firms in the market. A consistent pattern of this result emerges from the empirical works. Most of the studies found that the distribution of the cooperative links (agreements) is irregular and the development of the network is based on the existence of the highly connected firms (Riccaboni and Pammolli , 2002; Powell et al., 2005; Tomasello et al., 2013).

Finally, we explore the impact of the growth of the market size on the R&D investment.<sup>1</sup> When the goods are independent, the investments remain constant in spite of the market growth. However, when the goods are homogeneous, the effect of growing the market depends on the R&D network structure. The growth of the market positively affects the R&D investment if the density of the network is high; otherwise the R&D investment declines with growing the market size. This is in line with the empirical findings where most studies found that the cooperation exhibit characteristic features of complex networks (Tomasello et al., 2013; Ahuja , 2000; Stuart, 2000; Verspagen and Duysters , 2004).

The paper proceeds as follows. In the next section, we review some economic and networks issues and introduce the R&D model. In the third section, we provide the main study. In the fourth section, we conclude our study.

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<sup>1</sup> When we say size of the market or the network, we mean the number of firms.

## 2. LITERATURE REVIEW

### 2.1. R&D Network Model

A network is an ordered pair  $G(N, E)$  where  $N = \{i, j, k, \dots\}$  is a set of nodes connected by links  $E = \{ij, jk, \dots\}$  (Jackson, 2008). We consider undirected networks in the sense that each link between any two nodes runs in both directions. We also consider simple networks that have no parallel links (links that have the same end nodes) or loops (links where their start and end nodes are the same). For  $n$  nodes and  $m$  links, the density of the network is  $D = 2m/(n(n-1))$ . The set of nodes that are linked to node  $i$  is defined as a neighborset of node  $i$ :  $N_i = \{j \in N: ij \in E\}$ . The second order neighbors of node  $i$  is the set of all nodes sharing links with neighbors of node  $i$  such that those nodes are not linked to  $i$ :  $N_i^{(2)} = \bigcup_{j \in N_i} N_j \setminus \{i \cup N_i\}$ . The length of the first order neighbors' set of node  $i$  is a degree of that node (i.e.,  $|N_i| = \text{deg}(i)$ ). The network is called  $k$ -regular if  $\text{deg}(i) = k$  for each  $i \in N$ .

In the R&D network, nodes represent firms and links represent R&D agreements. We consider the R&D network model by Goyal and Moraga-Gonzalez (2001) where the cooperation is modeled as a three-stage game. In the first stage, firms choose their partners and the cooperating firms are joining together via links to form a network. The R&D spillover occurs between any two non-cooperating firms. In the second stage, firms choose their level of cost reducing R&D investment. In the third stage, firms compete by setting their products (Cournot competition).

In Goyal and Moraga-Gonzalez, the effective R&D investment for each firm is described by the following equation:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n, \quad (1)$$

where  $x_i > 0$  denotes R&D investment of firm  $i$ ,  $N_i$  is the set of firms participating in R&D with firm  $i$  and  $\beta \in [0,1]$  is the R&D spillover. The effective R&D investment reduces firm  $i$ 's marginal cost of production:

$$c_i = \bar{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \quad i = 1, \dots, n, \quad (2)$$

where  $\bar{c}$  is the marginal cost of the production.

### 2.2. Economic Model

We consider the inverse demand function given in the following equation:

$$D_i^{-1} = p_i = a - q_i - \lambda \sum_{j \neq i} q_j, \quad i = 1, \dots, n, \quad (3)$$

where  $a > 0$  denotes the willingness of consumers to pay and the parameters  $p_i$  and  $q_i$  are the price and quantity of good  $i$ , respectively. The parameter  $\lambda \in [-1, 1]$  is the differentiation degree where if  $\lambda < 0$  ( $\lambda > 0$ ), goods are complements (substitutes). In this paper, we consider the case when the goods are independent ( $\lambda = 0$ ) and homogeneous ( $\lambda = 1$ ).

The effort is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is  $\gamma x_i^2$ , where  $\gamma > 0$  D'Aspremont and Jacquemin (1988). The profit  $\pi_i$  for firm  $i$  is the difference between revenue and production cost minus the cost of R&D

$$\pi_i = (p_i - c_i)q_i = (a - q_i - \lambda \sum_{j \neq i} q_j - c_i)q_i - \gamma x_i^2, \quad (4)$$

where  $c_i$  is the production cost given by equation 2.

The total Welfare ( $TW$ ) is the total surplus of consumers plus the industry profit

$$TW = \frac{(1-\lambda)}{2} \sum_{i=1}^n q_i^2 + \frac{\lambda}{2} (\sum_{i=1}^n q_i)^2 + \sum_{i=1}^n \pi_i . \quad (5)$$

For the equilibrium, we assume that the marginal cost  $\bar{c}$  is constant and equal for all firms. We identify the subgame perfect Nash equilibrium by using backwards induction. Here, we show the reader how to calculate the equilibria and the final list of the equilibria is provided in the Appendix.

Under Cournot competition, we solve  $\frac{\partial \pi_i}{\partial q_i} = 0$  for any firm  $i$ . This yields the best response function of quantity of good  $i$ :

$$q_i = \frac{a - c_i - \lambda \sum_{j \neq i} q_j}{2} . \quad (6)$$

Substituting the best response functions (equation 6 for each  $i$ ) into each other yields the symmetric equilibrium that is Nash equilibrium for the production quantity:

$$q_i = \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)((n-1)\lambda + 2)} . \quad (7)$$

By substituting (7) into the profit function (4), the equilibrium profit is<sup>2</sup>

$$\pi_i = \left[ \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda \sum_{j \neq i} c_j}{(2-\lambda)((n-1)\lambda + 2)} \right]^2 - \gamma x_i^2 . \quad (8)$$

Calculating the equilibrium investment  $x_i$  depends on the structure of the R&D network. By knowing the structure, we find the cost function  $c_i$  to substitute it into the profit function (8). Then, we calculate the best response function of R&D investment for each firm  $i$  ( $\frac{\partial \pi_i}{\partial x_i} = 0$ ). By plugging them into each other, we have the symmetric equilibrium for the R&D investment.

Note that, the parameter  $\gamma > 0$  should be high to avoid negative outcomes. To have suitable values of  $\gamma$ , the effort and cost functions should be non-negative and the second order condition for maximizing profit function ( $\frac{\partial^2 \pi}{\partial x^2} > 0$ ) should be satisfied. According to Goyal and Moraga-Gonzalez, we have

$$\begin{cases} \gamma > \max\left\{\frac{an}{4\bar{c}}, \frac{n}{4}\right\} \text{ if } \lambda = 0 \\ \gamma > \max\left\{\frac{a}{4\bar{c}}, \frac{n^2}{(n+1)^2}\right\} \text{ if } \lambda = 1 \end{cases} \quad (9)$$

<sup>2</sup> Note that the equilibrium profit function can be expressed in a more convenient form for practical calculation:  $\pi_i^* = (q_i^*)^2$ .

Henceforth,  $\gamma_{\lambda_0}^*$  and  $\gamma_{\lambda_1}^*$  are suitable values for the equilibrium outcomes under independent and homogeneous goods, respectively.

### 3. THE OUTCOMES

The expenditure of firms on R&D is affected by several matters. Here, we consider some of these matters that can be divided into two major factors. The first factor is the market structure (i.e., the possible relationship between the products of firms). Therefore, as the substitution rate increases, the competition between firms increases and vice versa. The intensity of the competition between firms has an impact on the strategy of financing of R&D. The second factor is the R&D network structure. The growth of the R&D agreements leads to increase of the density of the network. This in turn affects the strategy of firms in investing in R&D.

#### 3.1. Factors Affecting Expenditure on R&D

The expenditure of firms on R&D is affected by several matters. Here, we consider some of these matters that can be divided into two major factors. The first factor is the market structure (i.e., the possible relationship between the products of firms). Therefore, as the substitution rate increases, the competition between firms increases and vice versa. The intensity of the competition between firms has an impact on the strategy of financing of R&D. The second factor is the R&D network structure. The growth of the R&D agreements leads to increase of the density of the network. This in turn affects the strategy of firms in investing in R&D.

##### 3.1.1. The Market Structure versus Financing Strategy

It is a matter for firms to know who is in a market, whether or not competitor. This does not only impact the strategic interaction of firms in the marketplace in terms of setting the product, but it also impacts the overall outcomes. Meaning that, the individual and social outcomes vary according to the product type of firms.

Comparing the expenditure of firms on R&D for different relationships of goods sheds light on the relation between the firms' position in the market and the R&D investment. In other words, the amount of the expenditure on R&D varies according to the market structure. When firms produce differentiated goods, the financing of R&D is massive compared to the case when they produce substitute goods. As a result of this, as the differentiation rate approaches the lowest (highest) value, the expenditure of firms on R&D reaches the maximum (minimum) amount.

The latter result can be proven for two firms in the industry. The following proposition states that there is a negative relationship between the expenditure of firms on R&D and the differentiation rate.

**Proposition 1** *Suppose a duopoly market where firms compete by setting the quantity.<sup>3</sup> When the differentiation degree approaches its minimum value, the R&D investment reaches the maximum amount and vice versa.*

The proof is presented in the Appendix.

In the following, we show the impact of the differentiation degree on the investment of firms in R&D for an oligopoly market.<sup>4</sup> We do this by comparing the optimal investment for independent and homogeneous goods under the network concept in absence of the R&D spillover.

**Proposition 2** *Suppose an oligopoly market where firms compete by setting the quantities and the R&D cooperation forms a regular network. The R&D investment of firms for independent goods is higher than for homogeneous goods.*

The proof is presented in the Appendix.

<sup>3</sup> A duopoly market means the market that consists only of two firms.

<sup>4</sup> An oligopoly market means the market that consists of more than two firms.

Figure 1 illustrates the previous result for ten firms in a market. Regardless of the network structure, the figure shows that the expenditure on R&D is higher if firms produce independent goods.

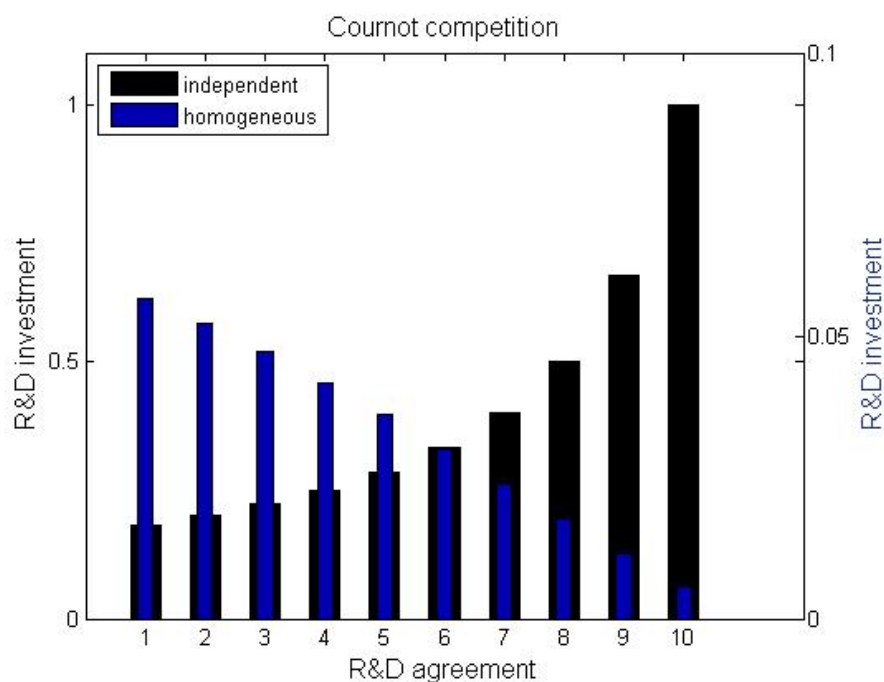
**Example 1** Suppose ten firms compete in a market by the quantities and the R&D cooperation forms a regular network. In the absence of the spillover, the equilibrium investment is plotted for  $a = 12$ ,  $\bar{c} = 10$  and  $\gamma = \max\{\gamma_{\lambda_0}^*, \gamma_{\lambda_1}^*\} = 3$ .

### 3.1.2. The Network Structure versus Financing Strategy

This section discusses the impact of the density of the cooperation on the strategy of the expenditure on R&D. This includes R&D spillovers resulting from the network structure of the R&D cooperation. In the previous section, we find that the rate of the R&D investment varies according to the product type. In this section, we show that the impact of the R&D agreements on the investment also varies according to the product type. The results presented in this section were stated by Goyal and Moraga-Gonzalez (2001).

On one hand, when firms produce differentiated goods, the investment of firms in R&D increases with the growing number of R&D agreements. Also, the spillover seems a positive factor in the R&D investment. In the sense that as the spillover increases, the expenditure of firms on R&D increases. On the other hand, when firms produce homogeneous goods, the R&D agreements negatively affect the R&D investment. Meaning that, when the R&D agreements increase, the expenditure on R&D decreases. The spillover in this case is not an encouraging factor for large firms to invest in R&D, but it appears especially important for small firms. Figures 2 and 3 illustrate these results related to the impact of the R&D agreements and the spillover on the strategy of the expenditure on R&D.

**Figure 1: Comparison between the Investments in R&D for Independent and Homogeneous Goods.**



Regardless of the number of R&D agreements, the expenditure of firms on R&D for independent goods is higher than for homogeneous goods. The outcomes are plotted for  $a = 12$ ,  $\bar{c} = 10$ , and  $\gamma^* = 3$ .

In addition, there is an indirect effect of R&D agreements on expenditure. In other words, when any two firms cooperate, how other firms' expenditure affected? The effect varies according to the market structure. If firms are in a weak competitive market, growing the neighborhood of any order raises the investment. This can be

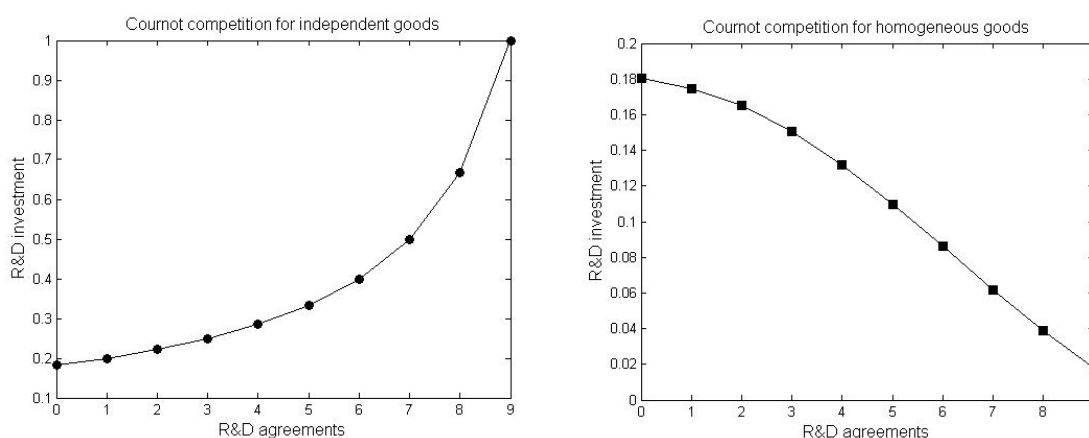
observed by comparing the expenditure of firm 1 in network  $G_3$  with that in network  $G_1$  after the other firms cooperate (see Example 2). However, in a competitive market, increasing the second order neighbors always reduces the investment in R&D.

**Example 2** Suppose an oligopoly market where firms compete by setting the quantities.

1) Suppose there are ten firms in a market where the R&D cooperation forms a regular network. In the absence of the spillover, Figure 2 shows the impact of the R&D agreements on the strategy of the expenditure on R&D.

2) Suppose there are three firms in a market where the R&D cooperation forms an irregular network. Figure 3 shows the possible R&D relationships and the impact of the R&D spillover on the investment in R&D.

**Figure 2: The Financing of R&D versus Growing the Agreements and Spillovers.**



If the products are independent, the financing of R&D increases with growing the R&D agreements. In contrast, if the products are homogeneous, the opposite occurs with growing the R&D agreements. The equilibria are plotted for  $a = 12$ ,  $\bar{c} = 10$ , and  $\gamma_{\lambda_0}^* = 3$  and  $\gamma_{\lambda_1}^* = 1$ .

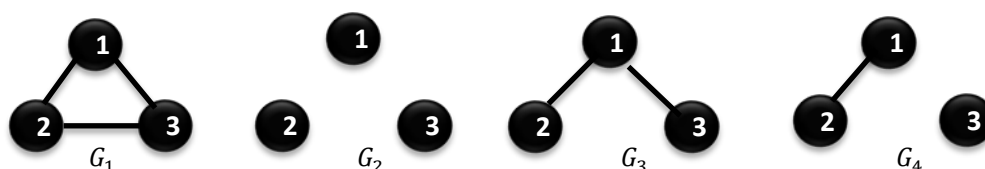
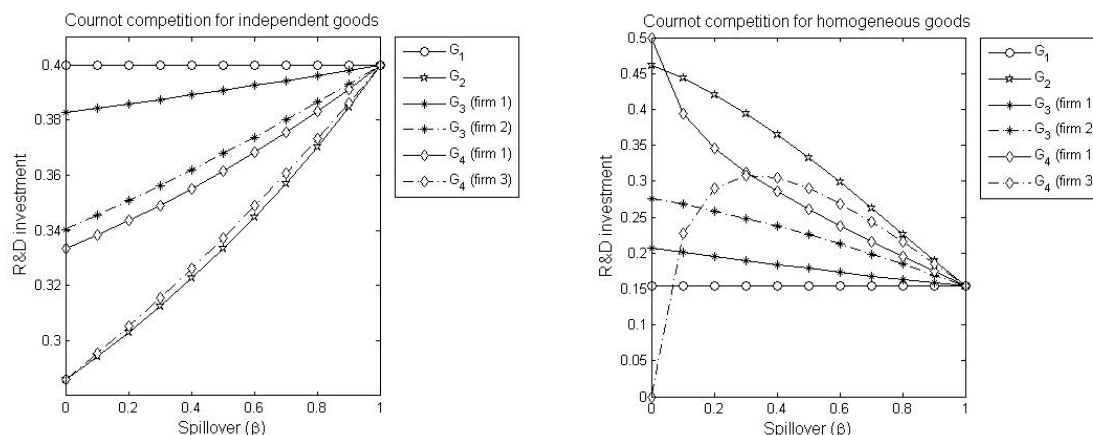


Figure 3: The Expenditure of Firms on R&D versus the R&D Spillover



Note that, in each of  $G_1$  and  $G_2$ , there is one group of firms, but the other networks, there are two groups. In the network  $G_3$ , there is hub (firm 1) and peripheries (firms 2 and 3) and in the network  $G_4$ , there are linked firms (firms 1 and 2) and an isolated firm (firm 3). The outcomes are plotted for  $a = 12$ ,  $\bar{c} = 10$  and  $\gamma_{\lambda_0}^* = 2$  and  $\gamma_{\lambda_1}^* = 1$ .

### 3.2. Motivation Behind Expenditure on R&D

From the R&D model (see Section 2.1), firms invest and cooperate in R&D to minimize the marginal cost of the production. This in turn maximizes the individual profit and enhance the social outcomes in somewhat. This implies that the change of the expenditure on R&D determined by the structure of the market and the network affects the economic variables.

Recall, the expenditure of firms on R&D is high when they produce very differentiated goods. Also, in this case, the expenditure is maximized when firms form a dense R&D network. We find that the significant increase in the investment positively reflects on other economic outcomes. The individual quantity and profit and the social welfare reach the maximum values when the dense R&D network consists of differentiated firms. In the case when firms produce homogeneous goods, the decrease of the expenditure on R&D with the number of the agreements does not generate low individual profits where there is always positive relation between the individual agreements and profits. For the overall outcomes, it seems that the positivity of the R&D agreements does not appear on the total welfare, particularly when R&D spillovers are high.

When comparing the equilibrium outcomes for independent and homogeneous goods, the results indicate that if goods are independent, the increase of the outcomes with the R&D agreements is high compared to the increase of the outcomes for the homogeneous goods.

**Proposition 3** Suppose an oligopoly market where firms compete by setting the quantities and the R&D cooperation forms a regular network. In the absence of the spillover, the equilibrium outcomes of the quantity, profit and total welfare for independent goods are higher than the equilibrium outcomes for homogeneous goods.

The proof is presented in the Appendix.



**Example 3** Suppose ten firms compete in a market by the quantities and the R&D cooperation forms a regular network such that the spillover is set zero. Figure 4 shows the quantity, profit and the total welfare for  $a = 12$ ,  $\bar{c} = 10$ ,  $\beta = 0$  and  $\gamma = \max\{\gamma_{\lambda_0}^*, \gamma_{\lambda_1}^*\} = 3$ .

### 3.3. Growth of Investors versus Expenditure on R&D

Increasing firms in the market leads to changes in the mathematical properties of the network and hence in the economic outcomes. In terms of the network, many empirical works have investigated the patterns through which R&D cooperation networks of worldwide firms develop and grow. Most of these studies found that the cooperation exhibit characteristic features of complex networks that describe many of the social networks (Tomasello et al., 2013; Ahuja, 2000; Stuart, 2000; Verspagen and Duysters, 2004).

In terms of economics, we want to examine the impact of new entering firms into the R&D network on the expenditure on R&D. We expect that the results depend on the market structure as the case of when growing the R&D agreements (see Section 3.1.2).

In this section, we deal with regular networks since the equilibria cannot be generalized if the distribution of the R&D agreements is asymmetric. Therefore, let  $G$  be an R&D network consists of  $n$  firms and assume that there are new firms want to enter the network. For each new firm entering the network, we assume that the result of the cooperation is  $k$ -regular networks.

The following proposition states that if goods are independent, the increase of firms in the network such that number of R&D agreements stays constant does not have an impact on the expenditure of firms on R&D. Meaning that, the investment of firms in R&D under any two different regular networks that have the same number of agreements is identical. In contrast, if goods are homogeneous, the impact of increasing firms in the network on the R&D expenditure depends on the number of agreements built among them. In a low dense network, the increase of firms negatively affects the R&D expenditure; whereas in a dense network, the increase of firms positively affects the R&D expenditure.

**Proposition 4** Suppose an oligopoly market where firms compete by setting the quantities and the R&D cooperation forms a regular network. Assume the network size grows where the number of R&D agreements is fixed. In the absence of the spillover,

1. if firms produce independent goods, the R&D expenditure stays constant.
2. if firms produce homogeneous goods and the density of the network is not low, the R&D expenditure increases.

The proof is presented in the Appendix.

**Example 4** Suppose an oligopoly market where firms compete by setting the quantities. Suppose that the threshold network consists of five firms and the network size is increased by one where with each new firm, the resulting network is regular. The graph shows the impact of the R&D agreements on the expenditure of firms on R&D for homogeneous goods where the spillovers between non-cooperating firms are zero.

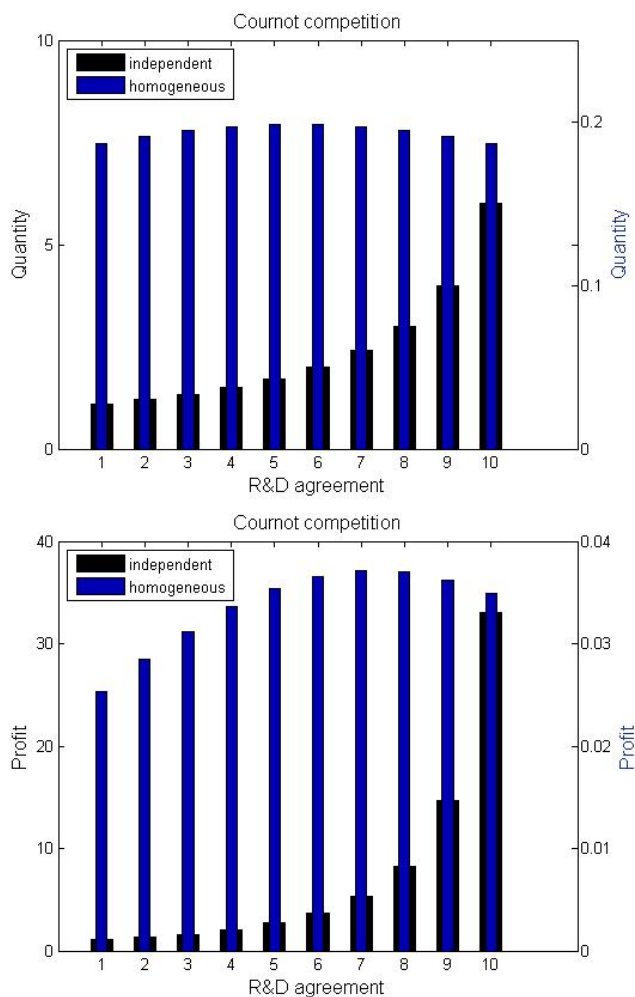
## 4. CONCLUSION

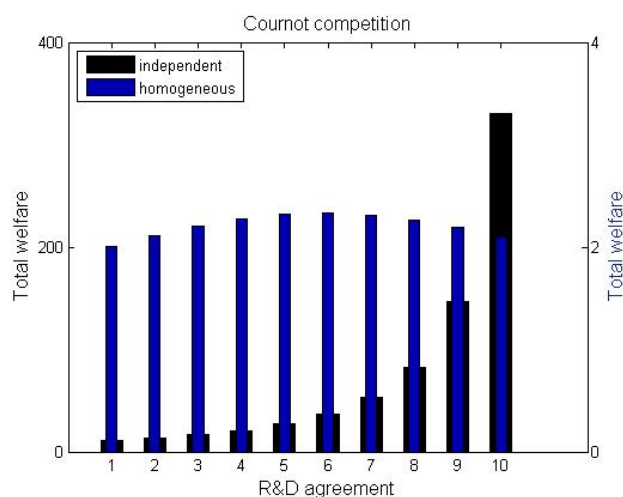
In this paper, we focused on the expenditure of firms on R&D. Firstly, we studied the impact of the market and network structure on the expenditure. The results suggested that the expenditure on R&D is significantly large if firms are in a highly differentiated market and in this case, forming a dense cooperation network improves the investment. Secondly, we discussed the role of the investment in R&D on the individual and social outcomes. It seems that the increase of the expenditure on R&D improves the outcomes. Also, the cooperation always leads to higher profits, regardless of the market structure. Finally, we studied the impact of the growth of the market size on the investment in R&D. If the cooperation rate is constant, the increase of new firms does not change the investment rate for independent goods. However, for homogeneous goods, the growing market size enhances the investment if the network density is not low.

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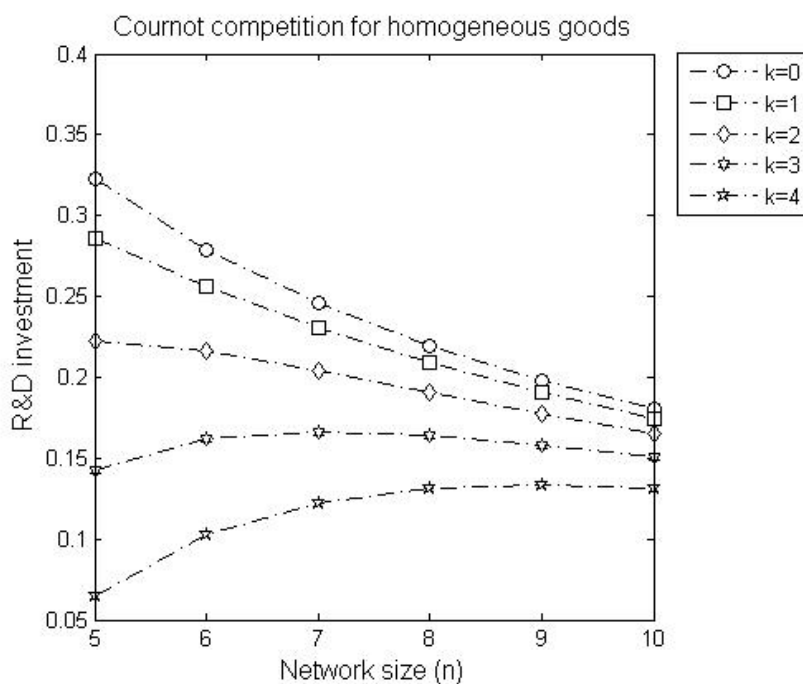
**Figure 4: Comparison of the Outcomes for Independent and Homogeneous Goods.**





The outcomes for independent goods are higher than for homogeneous goods, regardless of the number of R&D agreements. The outcomes are plotted for  $a = 12$ ,  $\bar{c} = 10$  and  $\gamma = 3$ .

Figure 5: Growth of Investors versus Expenditure on R&D for Homogeneous Goods



The parameters used to plot the graph are  $a = 12$ ,  $\bar{c} = 10$  and  $\gamma_{\lambda_1}^* = 1$ .

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## Appendix

### (A) Nash equilibria

#### Symmetric network

$$x_{\lambda_1}^* = \frac{(a-\bar{c})(n-k)}{\gamma(n+1)^2-(n-k)(k+1)} \quad (10 \text{ a})$$

$$q_{\lambda_1}^* = \frac{\gamma(a-\bar{c})(n+1)}{\gamma(n+1)^2-(n-k)(k+1)} \quad (10 \text{ b})$$

$$x_{\lambda_0}^* = \frac{(a-\bar{c})}{4\gamma-k-1} \quad (10 \text{ c})$$

$$q_{\lambda_0}^* = \frac{2\gamma(a-\bar{c})}{4\gamma-k-1} \quad (10 \text{ d})$$

#### Asymmetric network

$$x_{G_1} = \frac{(a-\bar{c})}{(4\lambda^2+8\lambda+4)\gamma-3} \quad (11 \text{ a})$$

$$q_{G_1} = \frac{2\gamma(a-\bar{c})(\lambda+1)}{(4\lambda^2+8\lambda+4)\gamma-3} \quad (11 \text{ b})$$

$$x_{G_2} = \frac{(a-\bar{c})(\lambda(2\beta-1)-2)}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3} \quad (12 \text{ a})$$

$$q_{G_2} = \frac{2\gamma(a-\bar{c})(\lambda^2-\lambda-2)}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3} \quad (12 \text{ b})$$

$$x_{G_3}(\text{firm 1}) = \frac{(a-\bar{c})(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2)}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(12\gamma^2-4(\beta+2)\gamma+\beta-1)} \quad (13 \text{ a})$$

$$q_{G_3}(\text{firm 1}) = \frac{2\gamma(a-\bar{c})(1+\lambda)(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2)}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(12\gamma^2-4(\beta+2)\gamma+\beta-1)} \quad (13 \text{ b})$$

$$x_{G_3}(\text{firm 2}) = \frac{2\gamma(a-\bar{c})((\beta\lambda-2)(1+\lambda)(\lambda-2))}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(12\gamma^2-4(\beta+2)\gamma+\beta-1)} \quad (13 \text{ c})$$

$$x_{G_3}(\text{firm 2}) = \frac{2\gamma^2(a-\bar{c})(\lambda-\lambda^2+2)^2}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(12\gamma^2-4(\beta+2)\gamma+\beta-1)} \quad (13 \text{ d})$$

$$x_{G_4}(\text{firm 1}) = \frac{(\beta\lambda-2)(a-\bar{c})(2\beta^2\lambda-3\beta\lambda-2\beta-2\gamma\lambda^3+6\gamma\lambda^2+\lambda-8\gamma+2)}{2(-4\gamma^2\lambda^6+12\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1))} \quad (14 \text{ a})$$

$$q_{G_4}(\text{firm 1}) = \frac{\gamma(a-\bar{c})(\lambda-\lambda^2+2)(3\beta\lambda-2\beta^2\lambda+2\beta+2\gamma\lambda^3-6\gamma\lambda^2-\lambda+8\gamma-2)}{2(-4\gamma^2\lambda^6+12\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1))} \quad (14 \text{ b})$$

$$x_{G_4}(\text{firm 3}) = \frac{(a-\bar{c})(\lambda-2\beta\lambda+2)(\beta\lambda-\beta^2\lambda+2\beta+\gamma\lambda^3-3\gamma\lambda^2+4\gamma-2)}{2(-4\gamma^2\lambda^6+12\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1))} \quad (14 \text{ c})$$

$$q_{G_4}(\text{firm 3}) = \frac{2\gamma(a-\bar{c})(\lambda-\lambda^2+2)(\beta\lambda-\beta^2\lambda+2\beta+\gamma\lambda^3-3\gamma\lambda^2+4\gamma-2)}{2(-4\gamma^2\lambda^6+12\gamma^2\lambda^6+12\gamma^2\lambda^5+S_4\lambda^4+S_5\lambda^3+S_6\lambda^2+S_7\lambda+4(8\gamma^2-6\gamma-\beta^2+1))} \quad (14 \text{ d})$$

where  $S_1 = 2(20\gamma^2 + (2\beta + 1)\gamma)$ ,  $S_2 = 2(4\gamma^2 + (2\beta^2 + 7)\gamma)$ ,  $S_3 = 64\gamma^2 + 4\beta(B - 1)(4\gamma - 1)$ ,  $S_4 = 12\gamma^2 + (6\beta^2 - 4\beta + 1)\gamma$ ,  $S_5 = -44\gamma^2 - (6\beta^2 + 12\beta - 3)\gamma$ ,  $S_6 = (6 + 24\beta - 12\beta^2)\gamma - 24\gamma^2 - \beta(\beta^2 - 1)(2\beta - 1)$ ,  $S_7 = 2(\beta(2\beta^2 - \beta - 3) + 24\gamma^2 - (10 - 16\beta)\gamma + 1)$ .

## (B) Proof of propositions

**Proof of Proposition 1.** For a duopoly market, the R&D investment of firms is given by the following equation

$$x^* = \frac{(a - \bar{c})(2 - \lambda\beta)}{\gamma(2 + \lambda)^2(2 - \lambda) - (1 + \beta)(2 - \lambda\beta)}.$$

The rate at which the value of the investment changes with respect to the change of the differentiation rate is

$$\frac{\partial x^*}{\partial \lambda} = \frac{-2\gamma(a - \bar{c})(2 + \lambda)((\lambda^2 - \lambda + 2)\beta + (2 - \lambda\beta))}{(\gamma(2 + \lambda)^2(2 - \lambda) - (1 + \beta)(2 - \lambda\beta))^2}.$$

Since  $\lambda^2 - \lambda + 2 > 0$  for all values of  $\lambda \in [-1, 1]$ , then  $\frac{\partial x^*}{\partial \lambda} < 0$ . This implies that the R&D investment decreases with increasing the differentiation rate. ■

**Proof of Proposition 2.** Assume that  $x_{\lambda_0}^*$  and  $x_{\lambda_1}^*$  are the R&D investments for independent and homogeneous goods, respectively. Thus, we want to prove that  $x_{\lambda_0}^* > x_{\lambda_1}^*$  where we choose  $\gamma = \max\{\gamma_{\lambda_0}^*, \gamma_{\lambda_1}^*\}$ .

$$x_{\lambda_0}^* - x_{\lambda_1}^* = \frac{\gamma(a - \bar{c})((n + 1)^2 - 4(n - k))}{(4\gamma - k - 1)(\gamma(n + 1)^2 - (n - k)(k + 1))}.$$

Since  $(n + 1)^2 > 4(n - k)$  where  $k \leq n - 1$ , then  $x_{\lambda_0}^* > x_{\lambda_1}^*$ . This means that the equilibrium investment for independent goods is larger than for homogeneous goods. ■

**Proof of Proposition 3.** Assume that  $q_{\lambda_0}^*$ ,  $\pi_{\lambda_0}^*$  and  $TW_{\lambda_0}^*$  are the quantity, profit and the total welfare for independent goods. Similarly, assume  $q_{\lambda_1}^*$ ,  $\pi_{\lambda_1}^*$  and  $TW_{\lambda_1}^*$  are equilibria for homogeneous goods. We want to prove that the equilibria for independent goods are larger than for homogeneous goods where  $\gamma = \max\{\gamma_{\lambda_0}^*, \gamma_{\lambda_1}^*\}$ .

For the equilibrium quantity, the required task is to show that  $q_{\lambda_0}^* > q_{\lambda_1}^*$  for any  $n$  and  $k$ . We find the difference between the two equilibria

$$q_{\lambda_0}^* - q_{\lambda_1}^* = \frac{\gamma(a - \bar{c})(2\gamma(n + 1)(n + 3) + (k + 1)(1 + 2k - n))}{(4\gamma - k - 1)(\gamma(n + 1)^2 - (n - k)(k + 1))}.$$

The expression  $1 + 2k - n < 0$  if  $k < (n - 1)/2$ , but in this case, we have  $2\gamma(n + 1)(n + 3) + (k + 1)(1 + 2k - n)$ . This implies  $q_{\lambda_0}^* > q_{\lambda_1}^*$  for any market size  $n$  and network structure  $k$ .

Now, we want to prove that  $\pi_{\lambda_0}^* > \pi_{\lambda_1}^*$  for any  $n$  and  $k$ . We find that

$$\text{sign}(\pi_{\lambda_0}^* - \pi_{\lambda_1}^*) = \text{sign}(\gamma(n + 1)^2(16\gamma^2 + (k + 1)(8\gamma - k - 1) + (n + 1)(4\gamma - 1)(2k - n + 1)) + (n - k)^2(k + 1)^2(4\gamma - 1)).$$

Note that all expressions are positive except  $(n + 1)(4\gamma - 1)(2k - n + 1) < 0$  if  $n > 2k + 1$ , but in this case  $\gamma$  is large (see the condition 9). This makes the expression on the right positive for any  $n$  and  $k$ . This implies  $\pi_{\lambda_0}^* > \pi_{\lambda_1}^*$ .

For the equilibrium total welfare, we are required to prove that for any  $n$  and  $k$ ,  $TW_{\lambda_0}^* > TW_{\lambda_1}^*$ . The proof is straightforward since total welfare is a combination of the quantity and profit (equation 5). We have proven that the last two equilibria are higher for independent goods. This implies  $TW_{\lambda_0}^* > TW_{\lambda_1}^*$  and then the result follows. ■

**Proof of Proposition 4.** We want to show that for independent goods, the R&D finance is affected by the industry size. For homogeneous goods, we show that the increase or decrease of the R&D finance with growing the industry size depends on the network structure.

For independent goods, the R&D finance is provided by the following equation:

$$x_{\lambda_0}^* = \frac{(a - \bar{c})}{4\gamma - k - 1}.$$

Thus,  $\frac{\partial x_{\lambda_0}^*}{\partial n} = 0$  and this implies that the R&D finance remains a constant amount when the market increases. For homogeneous goods, the R&D investment is

$$x_{\lambda_1}^* = \frac{(a - \bar{c})(n - k)}{\gamma(n + 1)^2 - (n - k)(k + 1)}.$$

To see the investment changes with respect to the change of the market size, we calculate  $\frac{\partial x_{\lambda_1}^*}{\partial n}$ . This yields

$$\frac{\partial x_{\lambda_1}^*}{\partial n} = \frac{\gamma(n + 1)(a - \bar{c})(2k - n + 1)}{(\gamma(n + 1)^2 - (n - k)(k + 1))^2}.$$

Now, the expression  $2k - n + 1 < 0$  if  $k < (n - 1)/2$  where  $0 \leq k \leq n - 1$ . This implies that in a low dense network, the increase of firms negatively affects the R&D finance; whereas in a dense network, the increase of firms positively affects the R&D finance. ■