



MODELING ISTANBUL STOCK EXCHANGE-100 DAILY STOCK RETURNS: A NONPARAMETRIC GARCH APPROACH

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ABSTRACT

Autoregressive conditional heteroscedasticity (ARCH) and Generalized ARCH (GARCH) models with various alternatives have been widely analyzed in the finance literature in order to model the volatility of the returns. In all of these models, the hidden variable volatility depends parametrically on lagged values of the process and lagged values of volatility (Bühlmann and McNeill, 2002) where the parameters are estimated with a nonlinear maximum likelihood function. In this paper a nonparametric approach to GARCH models proposed by Bühlmann and McNeill (2002) is followed to model the volatility of daily stock returns of the Istanbul Stock Exchange 100 (ISE 100) market from January 1991 to November 2012.

1. INTRODUCTION

In the empirical literature Bollerslev (1986)'s generalized autoregressive conditional heteroscedasticity (GARCH) models, which are the extensions of Engle (1982)'s autoregressive conditional heteroscedasticity (ARCH) models are the most widely used models both in the Turkish and international markets on modeling the stochastic stock return volatility since they have the capability of capturing volatility clusters. Many research has been done on analysis of volatility of common stocks traded in ISE and comparing the performance of different parametric volatility models especially the GARCH models regarding stock return volatility (Atakan, 2009), (Yalçın, 2007), (Özden, 2008), (Rüzgar and Kale, 2007), (Sevüktekin and Nargeleçekenler, 2006), (Kalaycı, 2005). It is well proposed that the volatility of ISE returns exhibits an autoregressive moving average procedure. Because of the dependency in financial returns, time varying conditional volatility models are more flexible approaches for modeling risk or for predicting the volatility of a stock portfolio.

In this paper we will use a nonparametric approach to GARCH models proposed by (Bühlmann and McNeill, 2002) in order to model the volatility of daily stock returns of ISE 100 market from January 1991 to November 2012 data. We apply this nonparametric approach which is less sensitive to model misspecification. We compare the estimation capability of nonparametric and parametric GARCH models on the volatility of ISE 100 returns for the period of January 1991 to November 2012.

There are many papers modelling the volatility of daily stock returns of ISE using parametric methods. Balaban, et al. (1996) have studied the daily volatility of ISE index return for the period of January 1989 to July 1995. They preferred to model the squared return by using autoregressive moving average (ARMA) procedure for forecasting the volatility. They proposed an AR (1) model which implies that the stock return fluctuation can be modeled by using past squared returns.

There are some other papers, which bring out that ISE return series has an ARCH effect. Balaban (1999) referred the advantages of ARCH models which are more reliable than moving averages models, for measuring the daily risk of ISE 100 for the period of 4 January 1988 to 31 December 1997. In a similar study by Korkmaz and Aydın (2002) exponentially weighted moving average (EWMA) and GARCH models are used to determine the VaR of ISE 30 return in the period of January 5, 1998 to January 31, 2002 and the results of the models are compared. Sarikovanlık (2006) estimates the volatility of the daily returns for 49 individual stocks in ISE between 1990-2005. In this study he concluded that GARCH (1,1) is the best model for modeling the variance of the stock returns. In her research, Sarioğlu (2006) investigates for the factors that affect the ISE stock market volatility in two periods January 1991-December 2004 and May 1996-December 2004. For two periods the study reveals same results that exchange rates, industrial manufacturing index, changes in the stock of money and foreign capital ratio are associated with ISE volatility. Moreover, effectiveness of GARCH models family is revealed and GARCH(1,1) model is selected as a fitted model on the index return volatility. Furthermore, Gökçe (2001) and Mazıbaş (2005) mentioned effectiveness of autoregressive conditional heteroscedastic models on volatility of several ISE indices. For the period of 1989-1997 Gökçe (2001) has studied on volatility of ISE 100 daily returns. Similar to Sarioğlu's (2006) paper results, in this study residual of the AR(1) model of daily returns follow GARCH(1,1) procedure. In another study by Mazıbaş (2005) for the period of 1997-2004, daily, weekly and monthly volatility in composite, financial, services and industry indices of ISE has been modeled by ordinary GARCH models and EGARCH, GJR-GARCH, Asymmetrical PARCH and CGARCH models. It has been concluded that weekly and monthly forecasts results are more precise than daily forecasts, due to the high volatility in daily returns. ARCH-type models has weakness for modeling daily volatility. Estimations have demonstrated the existence of asymmetry and leverage effects for all types of data for the period of 1989-1997.

In contrast to the results in these papers, Kılıç (2004) has analyzed the existence of long memory properties in daily ISE 100 for the period of 4 January 1988 to 23 October 2003 and he concluded that the AR(1)-FIGARCH model is the adequate model, which has evidence of long memory dynamics. In the research paper by DiSario et al. (2008) absolute values of daily ISE 100 returns are characterized by long memory as well as the squared and log squared values for the period of July 1988 to May 2004. They concluded that shocks to the stock index volatility decay slowly and in that period returns are associated with each other. However, forecasts of the conditional variance for a short period of time is more important than the long-run, since when investors consider holding the assets for only a certain period.

The motivation for using nonlinear models arises from the two properties of financial time series; firstly from the fact that the distribution of returns has heavy tails and is leptokurtic and secondly from the fact that the volatility of financial time series changes over time. Most of the papers on stock return data of ISE have proposed that stock returns have these properties. Particularly for financial data, nonparametric approaches for nonlinear time series have been developed.

Among many others, value-at-risk (VaR) approach is one of the most preferred nonparametric estimation methods to measure market risk. Modeling market risk is quite important for determining the variance of portfolio return in finance. After the global financial crisis, reporting portfolio risks have become an obligation by the regulators and value-at-risk is determined as a

standard way for measuring portfolio risk for the financial institutions. In this sense, to measure the credit risk of a portfolio, a risk manager considers the market risk which is the volatility of market index as a best reflector of the trend of a market. Since variance is the measure of risk and the dependency is the major feature of stock returns, it calls for a good model of variance that mimics the return distribution. Most generally, determining the distribution of returns is the main problem when estimating the market risk. This is because financial returns have a leptokurtic distribution with a dependency on the tails. Therefore, violation on the assumptions, such as having an identical distribution induces developers to seek a more general approach for modeling the risk. This is why historical value-at-risk (VaR) approach is preferred as a nonparametric estimation method to measure the market risk. According to Chen and Tang (2005), nonparametric estimators are free of distributional assumptions on return series and they can capture fat-tailed and asymmetric distribution of the return process.

Thus, all the applications on the return series of ISE Indices indicate that alternative approaches are needed for the ISE since it is an emerging market. Because of the use of high frequency asset return data, and restrictive and hard-to-estimate parametric models, flexible and computationally simple nonparametric approaches have been popular (Andersen et al., 2002). Bellini and Figa-Talamanca (2004) have studied on a nonparametric tool for measuring serial dependence in the tails of financial data. According to their research on many financial time series exhibit strong dependency for daily log returns which is not fully covered by usual GARCH models (Bellini and Figa-Talamanca, 2004).

Therefore, we use a different nonparametric approach to model the return volatility process of Turkish market risk. We utilize the nonparametric approach to GARCH models proposed by (Bühlmann and McNeill, 2002) in order to model the volatility of daily stock returns of ISE 100 market from January 1991 to November 2012 data. This nonparametric approach which is an iterative process using a nonparametric smoothing technique is less sensitive to model misspecification. After modeling the volatility with this method, the estimation capability of nonparametric and parametric GARCH models on the volatility of ISE 100 returns are compared for the period of January 1991 to November 2012. The rest of the paper is organized as follows: In section 2 parametric ARCH-GARCH and nonparametric GARCH models are described briefly. The data set used in the analysis is presented in section 3 and the parametric ARCH and GARCH results are given in this section. In section 4 nonparametric GARCH models are estimated and the results are summarized with a comparison of those obtained from parametric and nonparametric methods. Finally in section 5 the paper is concluded.

2. CONDITIONAL HETEROSCEDASTIC MODELS FOR STOCK RETURNS

Since the main concern in the stock exchange market is to model the volatility of the stock returns, it is crucial to obtain the stock returns. The simple returns are calculated as follows where P_t is the price of the stock at time t :

$$R_t = \frac{P_t}{P_{t-1}} - 1 \quad (1)$$

The natural logarithm of the simple return is called the continuously compounded return or log return calculated as given in Equation (2) (Tsay, 2002, pp.4):

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} \quad (2)$$

It is commonly assumed that the log returns r_t is independent and identically (iid) distributed as normal distribution with mean μ and variance σ^2 (Tsay, 2002, pp. 11). Therefore if it is believed that the stock returns contain a conditional volatility, this has to be modeled. In order to model the conditional volatility traditional ARCH, GARCH methods will be estimated and with the nonparametric GARCH approach the performance of the estimates is targeted to be improved.

Since Engle (1982)'s seminal work of ARCH, a wide range of volatility methods have been developed and applied in many areas such as risk management, portfolio management, option pricing, foreign exchange, interest rates etc. A summary for these models can be seen from (Wei, et al., 2010).

ARCH models were developed by Engle (1982) in order to model the volatility of time series, specifically the UK inflationary uncertainty. However, ARCH models have been widely used in many areas of economics and finance in analyzing the time-varying volatility. It is believed that the largest variation might happen around the peaks which more precisely implies that the probability of obtaining a large variation is greater than a small variation. An ARCH (q) model of a time series r_t is defined by (Engle, 1982) assuming that the mean of r_t is given as,

$$r_t | F_{t-1} \sim N(\mathbf{x}'_t \beta, \sigma_t^2) r_t = \mathbf{x}'_t \beta + \varepsilon_t \varepsilon_t = \sigma_t Z_t \quad (3)$$

where F_{t-1} refers to the information set available at time $t - 1$ where Z_t is a sequence of iid random variables with mean zero and variance one. Here the \mathbf{x}'_t may include lagged dependent and exogenous variables. Therefore the conditional variance,

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 \quad (4)$$

is defined as an explicit function of the q lagged squares of ε_t 's and $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$ (Tsay, 2002, pp. 82-83, 87). For the log-return series (r_t) a simple way to build an ARCH model is firstly to estimate an Autoregressive Moving Average (ARMA) model in order to remove any linear dependence in the data which is equivalent to removing the sample mean from the data. This should be done if the sample mean is significantly different from zero. Secondly, the residuals of the ARMA model are used to test for ARCH effects. Therefore, residuals ε_t obtained from the ARMA model, namely the mean corrected stock returns are squared and the squared residuals ε_t^2 are used to check for conditional heteroscedasticity. There are Ljung-Box test and Lagrange multiplier tests of Engle (1982) available for conditional heteroscedasticity. If the test results are significant then one can build an ARCH model. Thirdly, the order of the ARCH model (q) is determined according to the partial autocorrelation function (PACF) of the squared residuals of the ARMA model (ε_t^2) (Tsay, 2002, pp. 86-87). The order (q) is the number of the significant partial autocorrelation (PAC) values. Finally, the ARCH(q) model is estimated and checked for the significance of the coefficients.

In the Generalized autoregressive conditional heteroscedastic (GARCH) models developed by Bollerslev (1986), conditional variance is described in terms of weighted averages of past conditional variances and squared past returns as an extension to Engle (1982) ARCH models. A GARCH(p, q) model of the log-return series (r_t) with a suitable ARMA(l) model for the mean can be defined as

$$r_t = \sum_{i=1}^l a_i r_{t-i} + \varepsilon_t \varepsilon_t = \sigma_t Z_t \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \dots + \beta_p \sigma_{t-p}^2$$

where Z_t is a sequence of iid random variables with mean zero and variance one, $\alpha_0 > 0$ and $\alpha_i, \beta_i \geq 0$ for $i > 0$ (Tsay, 2002, pp. 82-83, 87).

In their paper, Tjøstheim and Auestad (1994) worked the possibility of identifying nonlinear time series models using nonparametric methods. Härdle and Chen (1995) present a selective review of the approaches that based on nonparametric model building procedure in time series analysis. They point that nonlinear and nonparametric time series analysis is useful in order to deal with the limitations of the ARMA models with constant mean. Härdle, et al., (1997) review some developments in modern nonparametric techniques for time series analysis. Engle and Gonzalez-Rivera (1991) addresses semi-parametric ARCH model by introducing a more efficient estimator based on a nonparametric estimated density. They also evaluate the loss of efficiency of the quasi-maximum likelihood estimator, which falsely assumes normality.

Bühlmann and McNeil (2002) proposed a nonparametric approach to GARCH modeling. Hou and Suardi (2012) considered Bühlmann and McNeil (2002)'s nonparametric approach to model and forecast crude oil price return volatility. They use 4845 daily observations on crude oil spot prices from West Texas Intermediate, from 6 January 1992 to 30 July 2010, in their application. According to their results on forecasting accuracy, the nonparametric GARCH model has superior performance to parametric GARCH models. They prefer their nonparametric approaches because of the non normality of distribution of oil prices. There are other papers on forecasting volatility of crude oil markets. Similar results that refer to effectiveness of GARCH models and extensions of GARCH models in oil market to ISE have been concluded in the applications respectively (Wei, et al., 2010).

In the next section we briefly introduce Bühlmann and McNeil (2002)'s nonparametric GARCH model before applying the method on ISE 100 returns.

2.1 Nonparametric GARCH Models

Bühlmann and McNeil (2002) proposed an algorithm for fitting the nonparametric GARCH models of first order (Bühlmann and McNeill, 2002). They have observed whether the nonparametric models give better estimates of the volatility process than parametric ones with the GARCH models and suggested the new estimator to compute dynamically changing measure of risk. In this paper, we apply this nonparametric method that does not require the specification of the functional form of the volatility and that does not regard to the distributional form of the innovation distribution.

Moreover, nonparametric GARCH models allow the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure. In this paper the stationary stochastic process $\{\varepsilon_t; 1 \leq t \leq n\}$ has the nonparametric GARCH(1,1) form given in (Bühlmann and McNeill, 2002):

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t \\ \sigma_t^2 &= f(\varepsilon_{t-1}, \sigma_{t-1}^2) \end{aligned} \tag{6}$$

In the nonparametric GARCH approach the exact form of $f: \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is unspecified and is estimated using a bivariate nonparametric smoothing technique which is less sensitive to model misspecification such as neglected asymmetric volatility (Bühlmann and McNeill, 2002, s. 666).

Assuming that $\{\varepsilon_t; 1 \leq t \leq n\}$ coming from a process satisfying (5), the estimation of a nonparametric GARCH model is applied with the following steps as proposed in (Bühlmann and McNeill, 2002):

1. Firstly, at the $m=0$ step, an estimate of volatility $\{\hat{\sigma}_{t,0}^2; 1 \leq t \leq n\}$ is obtained by fitting an ordinary parametric GARCH(1,1). Then the predictions from the GARCH(1,1) model are extracted which gives the $\hat{\sigma}_{t,0}^2; 1 \leq t \leq n$ estimates for the $m=0$ step of the algorithm. Since the first value is not estimated in returns, it is set as equal to the mean.
2. In the $m=1$ step, ε_t^2 is regressed with a nonparametric smoothing technique against ε_{t-1} and $\hat{\sigma}_{t-1,0}^2$ which are obtained from the parametric GARCH(1,1). The squared values of the residuals are obtained from the ARIMA model and the lagged values are the first lag of the residuals of the ARIMA model. The estimated variance of the return series is obtained from the previous step of the algorithm.
3. At the m 'th step, the algorithm is repeated and the $\hat{\sigma}_{t,m}^2$ is estimated by ε_{t-1} and $\hat{\sigma}_{t-1,m-1}^2$.

To compare the performance and the accuracy of the volatility estimates both an average squared estimation error and an average absolute estimation error for each iteration of the method are calculated. These performance measures are given as follows:

$$\begin{aligned}
 MSE(\hat{\sigma}_{.,m}) &= \frac{1}{n-r} \sum_{t=r+1}^n (\hat{\sigma}_{t,m} - \sigma_t)^2 \\
 MAE(\hat{\sigma}_{.,m}) &= \frac{1}{n-r} \sum_{t=r+1}^n |\hat{\sigma}_{t,m} - \sigma_t|
 \end{aligned}
 \tag{7}$$

With the nonparametric approach, it is expected to obtain a substantial improvement on the parametric estimates.

Another important reason in the development of nonparametric models is the lag selection procedure. The usual nonparametric models have less than satisfactory performance when dealing with more than one lag especially in the curse of dimensionality case. Alternative lag selection criteria have been studied for nonlinear autoregressive processes. Auestad and Tjøstheim (1990) and Tjøstheim and Auestad (1994) mention heteroscedasticity in financial returns and propose to use a nonparametric version of the final prediction error (FPE). Tschernig and Yang (2000) derived a nonparametric version of the Final Prediction Error for lag selection in nonlinear autoregressive time series under very general conditions including heteroscedasticity. Yang, et al. (1999) introduce a new nonparametric auto regression with multiplicative volatility and additive mean to have better estimations. Also Wang et al. (2012) proposed a new efficient semi-parametric GARCH modeling of volatility by taking account lag selection procedure.

3. DATA AND PARAMETRIC GARCH ESTIMATES

The data used in this paper is the daily stock returns of ISE100 market from November 1991 to November 2012. Daily stock returns are calculated using the Equation given in (2). All the calculations and estimations are done in R Statistical Environment (R, 2008). For the ARIMA and GARCH models, an R package by Pfaff and Stigler (2011) called “urca” is used (Pfaff and Stigler, 2011). The graphical representation of daily logarithmic returns is given in Figure 1 and the summary statistics are displayed in Table 1.

Figure 1: ISE 100 Daily Logarithmic Stock Returns 1991:1-2012:11 (logR91)

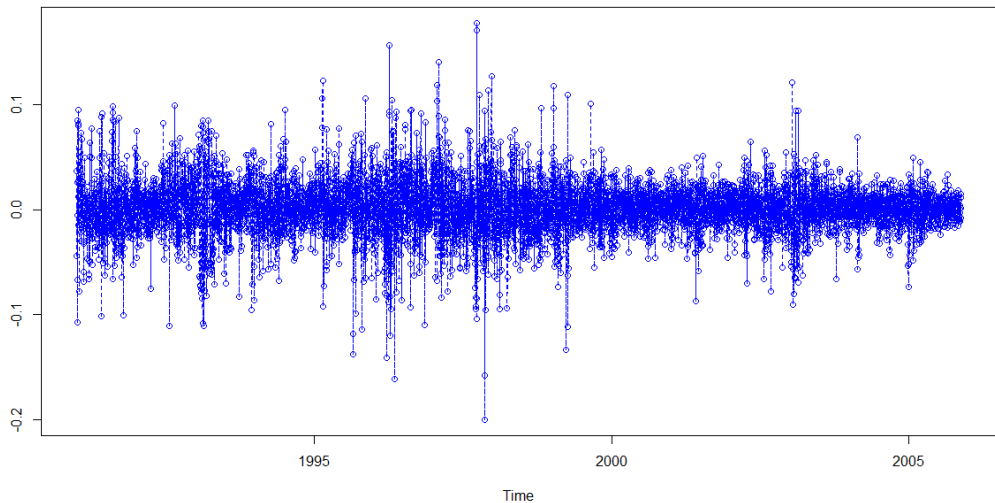
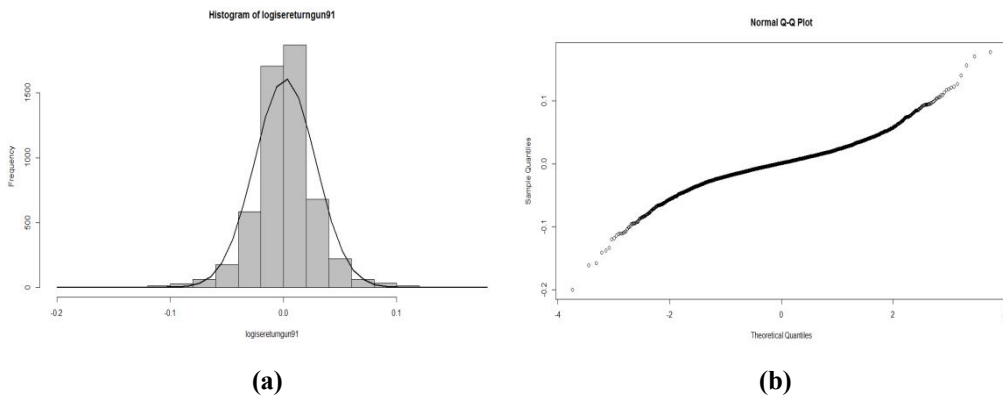


Figure 2: Histogram (a) and Q-Q Plot (b) of ISE 100 Daily Logarithmic Stock Returns 1991:1-2012:11 (logR91)



In Figure 2, we plot the histogram of ISE100 logarithmic returns over the period of 1991:1-2012:11 along with the normal curve. We see that this distribution is peaked and fat-tailed relative to the normal distribution. The Q-Q plot on the right side of Figure 2 shows that the tails of the ISE 100 returns are extreme relative to the normal distribution. Fat tails and peak distributions indicate that variances differ along time. As a result the volatility is not staying constant.

Table 1: Summary statistics for the logR91

	logR91
Mean	0.001418
Median	0.001420
Maximum	0.177736
Minimum	-0.199785
Std. Dev.	0.026955
Skewness	-0.034534
Kurtosis	10.217549
Jarque-Bera	4027.775
Probability	0.000000
Observations	5433
Test of the Mean ($H_0 : \mu=0$)	3.876266 (0.0001)

As it is seen from both Figure 2 and the summary statistics table in Table 1, the stock returns are not normally distributed and the kurtosis is high with an excess kurtosis value of approximately 10. Moreover, Jarque-Bera normality test (Trapletti et al., 2012) indicates that the logarithmic daily returns are not following a normal distribution. Therefore it is believed that the stock returns contain a conditional volatility.

In order to build an ARCH model, firstly any linear dependence in the daily log stock returns of manufacturing sector in ISE is removed. This is done by estimating an ARMA model with Maximum Likelihood estimation. Before performing an ARMA model the time series data is tested against stationarity. In most of the time series analysis methods the first step is to find out if the series is stationary or not. A time series (r_t) is said to be strictly stationary if the joint distribution of $(r_{t-1}, \dots, r_{t-k})$ is invariant under time shift (Tsay, 2002, pp. 23). On the other hand, a time series is weakly stationary if both the mean of r_t and the covariance between r_t and r_{t-1} are time-invariant, where 1 is an arbitrary integer (Tsay, 2002, pp. 23). In the finance literature, it is common to assume that return series is weakly stationary (Tsay, 2002, pp. 23).

The basic stationarity examination is to plot the time series so that it could be examined from a graph if the series has a trend or not. Though, a more precise way of exploring the stationarity of the series is applying unit root tests. Here we employed Augmented Dickey Fuller (ADF) and Philips Perron (PP) unit root tests and the results are provided in Table 2.

Table 2: ADF and Philips Perron (PP) Unit Root Test Results

	ADF Test Statistics			PP Test Statistics	
	Lag	None	Trend & Intercept	Intercept	Trend & Intercept
LogR91	1	-68.62963**	-68.84677**	-68.62888**	-68.84661**

*, ** show the significant values at 5% and 1%, respectively.

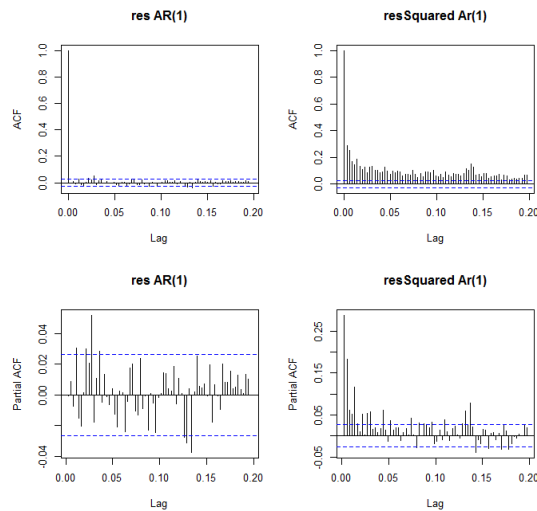
Here, we see that according to both of the unit root test results we can reject the null hypothesis that says there is a unit root. Therefore it is concluded that the logarithm of the stock returns is stationary at level.

After having seen that the returns are stationary at level, an ARMA model is built for modeling the average returns since an ARCH effect can be examined in a data that has zero mean. From the descriptive statistics table we see that the mean of the logarithmic returns is 0.001418 in the period of analysis. This mean is significantly different than zero¹. Therefore the average returns should be modeled in order to obtain a zero mean residuals. We have fitted an ARMA(1,0) model (which can be briefly called as AR(1) model) having observed both the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots. The AR(1) model is estimated as follows:

$$\begin{aligned} \text{Logreturns91} \quad \hat{r}_t &= 0.001427 + 0.068757 r_{t-1} && (8) \\ \text{t values} &[3.641] [5.080] \\ \text{p values} &(0.0003) (0.0000) \end{aligned}$$

After building the model, the residuals are obtained which have a zero mean (with a value of 0.000022136). Finally, an ARCH model is generated using these residuals. To identify if there is a need for an ARCH model the ACF and PACF of the squared residuals given in Figure 3 should be examined as well testing the squared residuals for conditional heteroscedasticity. The ACF and PACF clearly show the existence of conditional heteroscedasticity since there are no significant AC values in the ACF plot of the residuals of the AR(1) model. On the other hand, there are many significant AC values in the squared residuals of the AR(1) model.

Figure 3: Sample ACF and PACF of the squared residuals: (a) ACF of the squared residuals, (b) PACF of the squared residuals (lower left)



¹Significance value for a one sample t test with a test value of zero is 0.0001.

Apart from observing the ACF and PACF of the squared residuals of the ARMA model, Ljung-Box test could be applied to the squared residuals of the ARMA model to check for the conditional heteroscedasticity (Graves, 2012). The results are summarized in Table 3.

Table 3: Box-Ljung test for the residuals of AR(1) model

		Test Statistic	df	p-value
LogR91	Box-Ljung test	1771.707**	12	0.0000
	ArchTest	409.6759**	12	0.0000

*, ** show the significant values at 5% and 1%, respectively.

The output of the Ljung-Box test given in Table 3 and the examination of the ACF and PACF for various functions of the residuals indicate that there is conditional heteroscedasticity effect. Therefore GARCH model estimation is clearly necessary.

Following the literature that has well documented that ISE100 stock returns are very well modeled using a GARCH(1,1) model, we use a GARCH(1,1) parametric approach to estimate the first step volatility values. Table 4 provides the estimates and the significance levels of the estimates. It is clearly seen that the GARCH(1,1) estimates are significant.

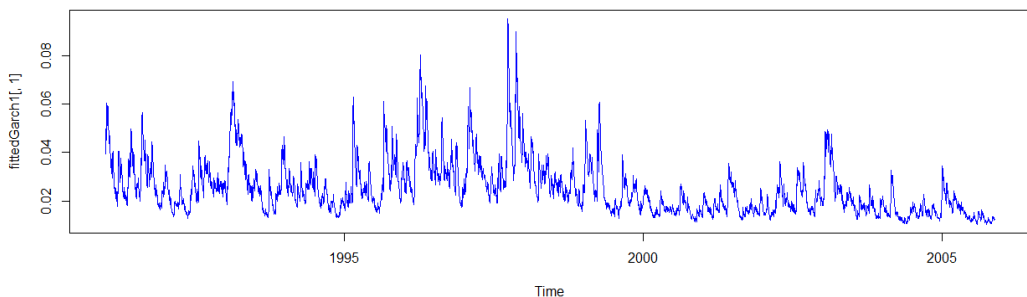
Table 4: GARCH(1,1) Estimates

		Estimate	Std. Error	t value	Pr(> t)
LogR91	a0	0.000007	0.000001	6.740268	0.0000**
	Resid^2	0.105703	0.005862	18.03295	0.0000**
	Garch	0.888319	0.005343	166.2521	0.0000**
Jarque-Bera test statistic:696.9948(d.f.:2), p-value: 0.0000					
ARCH-LM Ftest statistic: 7.62061 (d.f.: 1,5430), p-value: 0.0058					

It is also explicit that the residuals of GARCH(1,1) is not normally distributed and that they have ARCH effect in them.

The volatility estimates obtained from the parametric GARCH(1,1) model is displayed in Figure 4.

Figure 4: Parametric GARCH(1,1) Volatility Estimates

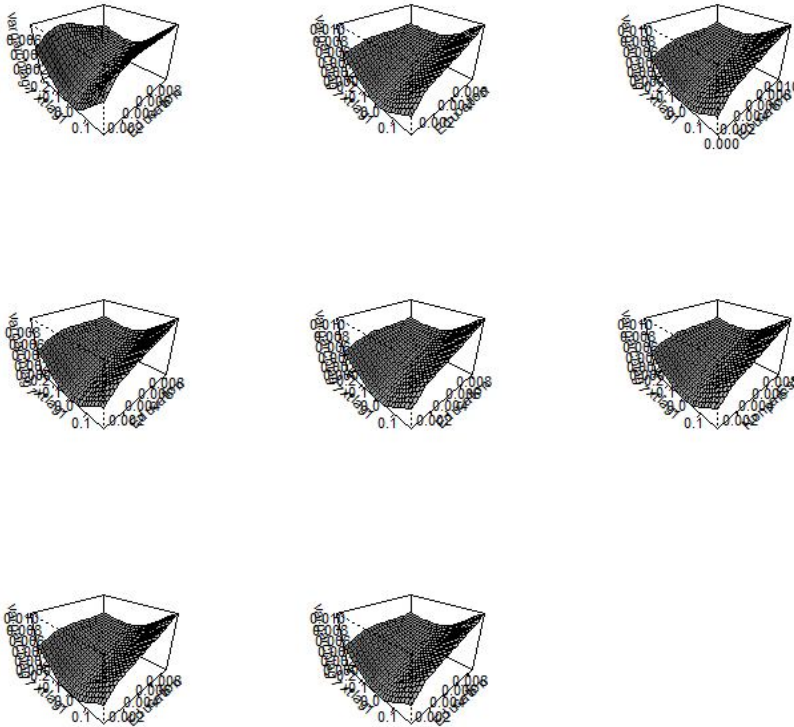


It is evident that volatility moves through time. In the next section we will use a nonparametric approach to estimate the volatility. The estimates obtained from the parametric GARCH(1,1) will be used as a starting point for the nonparametric process.

4. ESTIMATION OF A NONPARAMETRIC GARCH MODEL FOR THE ISE MARKET

Following the steps given in the previous section, the iterative smoothing process based on 7 iterations is applied in R using the default Loess function. The reason we stopped the iterative process at 7 was that the MSE and MAE measures were almost the same after a few iterations. The graphical output of the estimated surfaces obtained for the nonparametric GARCH method could be seen from Figure 5.

Figure 5: Nonparametric GARCH(1,1) estimates



The first estimated surface at the top left of Figure 5 belongs to the parametric GARCH(1,1) estimation and we see that once the nonparametric smoothing technique is applied the volatility surface is getting smoother and capturing the real volatility better. In order to compare the parametric and nonparametric results we calculated MSE and MAE terms at each iteration and these measures are given in Table 5.

Table 5: MSE and MAE measures of Nonparametric GARCH(1,1) in 7 steps

Step	MSE	MAE
Garch(1,1)	0.000346	0.014119
m=1	0.000335	0.013968
m=2	0.000336	0.014022
m=3	0.000334	0.013984
m=4	0.000333	0.013981
m=5	0.000334	0.013991
m=6	0.000334	0.013991
m=7	0.000333	0.013956

When we look at the mean errors we see that the largest improvement in the error figures is obtained mainly at first iteration. After the first iteration there is very little improvement. We can conclude that we obtained a sufficient improvement in the estimation of volatility even with only one step iteration using a nonparametric smoothing technique.

5. CONCLUSION

In this paper we model the volatility of daily stock returns of ISE 100 market from January 1991 to November 2012 data using the nonparametric approach to GARCH models proposed by (Bühlmann and McNeill, 2002). Many researches have been done on the analysis of volatility of ISE stock returns. When we look at the empirical literature review we see that GARCH(1,1) is found to be the best model for modeling the variance of the stock returns. Though most of the papers on stock return data of ISE have proposed that stock returns have the following properties: firstly, the distribution of returns has heavy tails and is leptokurtic and secondly, the volatility of financial time series changes over time. Using parametric methods when the returns have these properties can result in misleading conclusions. Therefore there is a need for an alternative method that is free of distributional assumptions on return series and that can capture fat-tailed and asymmetric distribution of the return process. Among the many alternatives, flexible and computationally simple nonparametric estimators are successful from this point of view and have been popular.

With this reason, Bühlmann and McNeil (2002) proposed an algorithm for fitting the nonparametric GARCH models of first order (Bühlmann and McNeill, 2002). It is well documented in their paper that the nonparametric models give better estimates of the volatility process than parametric ones with the GARCH models. In this paper, we applied this nonparametric method to ISE 100 daily stock returns. This is an iterative smoothing process based on 7 iterations which was applied in R using the default Loess function. In order to find the level of improvement we calculated the mean squared and absolute errors for both the parametric GARCH(1,1) and nonparametric GARCH(1,1). We observed an improvement in the errors of the estimations obtained with the nonparametric version even at first step of the iteration process.

In conclusion, we can easily say that when the distribution of the stock returns is unknown or has heavy tails and is leptokurtic, we can use the nonparametric volatility estimation method developed by Bühlmann and McNeil (2002), which is based on an iterative nonparametric process. Moreover, higher levels of GARCH model scan be investigated by this nonparametric method. The reason we have only applied a GARCH(1,1) nonparametric approach is that it is well documented in the literature that the volatility of ISE100 returns follow a GARCH(1,1) process.

Regarding our conclusion on ISE 100 return data, we have consistent results with the similar papers that apply this method. As a final note, referring to the effectiveness of nonparametric GARCH models for the univariate case, the multivariate nonparametric version of this approach could be developed for multivariate GARCH models.

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