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APPROXIMATE RULES TO CALCULATE MONTHLY MORTGAGE PAYMENTS

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KEYWORDS

Mortgage payment, mortgage interest rate, approximate rule.

ABSTRACT

There are many tools to calculate the monthly mortgage payment. If, however, any of these tools is not immediately available, it may not be easy to calculate the monthly mortgage payment. We propose three approximate rules for two popular 15- and 30-year mortgage terms. These rules work very well for historical mortgage interest rates that range from 4% to 15%. Not only financial professionals but also academicians can use them very easily in any informal situation without regard to availability of specific tools.

1. INTRODUCTION

Is there any easy way to calculate the monthly mortgage payment for a 15-year or a 30-year fixed rate mortgage loan? Actually, there are useful tools to calculate the monthly mortgage payment such as formula, financial calculator, and Excel. Online mortgage calculators are also offered by tons of websites. Any of these tools, however, may not be the answer for the question unless they are immediately available. More often than not, even financial professionals do not easily memorize the exact formula of the monthly mortgage payment. Even if they do, what if only a simple calculator is available? Unfortunately, it seems to be a difficult situation because there are few approximate rules known to people that can be easily used.

There are many studies that delve into the optimal decision of the mortgage term. Stansell and Millar (1976) investigate whether the variable rate mortgage payment significantly increases relative to the net income during a high inflation period and find that the variable payment does not impose a substantial burden on the mortgagor. Focusing on the same inflation effect, Barney and White (1986) shows that uncertainty in the future inflation can force even individuals with rising income to prefer the fixed payment mortgage to the graduated payment mortgage. As a similar study, MacDonald and Winson-Geidman (2012) find that inflation uncertainty decreases the adjustable rate mortgage (ARM) originations, specially, subprime ARM originations. Dhillon, Shilling, and Sirmans (1990) explain what mortgage term is preferred on the basis of wealth, tax, interest rate, and real housing price. Kistner (1998), Goff and Cox (1998), and Tomlinson (2002) support the long-term mortgage under some specific conditions. Gallay (2005) presents the mortgage decision based on scenarios using investment returns. Coulibaly and Li (2006) find that households eventually increase financial savings after the last mortgage payment. Basciano, Grayson, and Walton (2006) and Baek and Bilbeisi (2011) employ the Monte Carlo simulation to compare long-term and short-term mortgages in terms of their net gains. Deritis, Kuo, and Liang

(2010) formulate the impact of payment shock on mortgage performance and show that the payment shock is different depending on the delinquency situation of the loan and has the most impact on current loans. None of these studies, however, are directly associated with the mortgage payment method. The focus of our study is on developing approximate payment rules for the first time.

We propose three approximate rules that result from a simple linear regression. They are developed for two popular 15- and 30-year mortgage terms. In most cases, their percentage errors are less than 3%. In addition, they work for any amount of the mortgage loan.

Though these approximate rules are based on a very simple mathematical idea, it provides a really easy way to calculate the approximate mortgage payment in any informal situation regardless of availability of any specific tools.

2. APPROXIMATE RULES

2.1. 492+59 Rule (15-Year Mortgage Term)

Consider calculating the monthly mortgage payment for a 15-year, $200,000 mortgage loan at 4.50%. According to the exact formula,

Exact monthly payment = $(200{,}000 \times .045/12)/[1 - (1 + .045/12)^{-180}]$ = $1,529.99

The 492+59 approximate rule is proposed.

Approximate monthly payment for a 15-year mortgage loan = $(492 + 59 \times i) \times P$ (1)

where i is an interest rate (%), and P is a loan amount per $100,000. Using this approximate rule,

Approximate monthly payment = $(492 + 59 \times 4.5) \times 2$ = $1,515.

The difference between the exact monthly payment and the approximate monthly payment is $14.99 which is only 0.98% error. If the mortgage rate increases to 6.75%,

Exact monthly payment = $(200{,}000 \times .0675/12)/[1 - (1 + .0675/12)^{-180}]$ = $1,769.82

Approximate monthly payment = $(492 + 59 \times 6.75) \times 2$ = $1,780.50

The difference is $$10.68 which is only 0.6% error. With a mortgage rate of 5.60%, both payments are almost equal. Even when the amount of the mortgage loan increases or decreases, given a specific mortgage rate, the percentage error is always the same. For instance, even if the amount of the mortgage loan increases to $1,000,000 with the same rate of 6.75%, the percentage error is exactly equal to 0.6%.

2.2 179+71 Rule (30-Year Mortgage Term)

In the same way, the 179+71 approximate rule for a 30-year mortgage loan is proposed.

Approximate monthly payment for a 30-year mortgage loan = $(179 + 71 \times i) \times P$ (2)

On a 30-year, $350,000 mortgage loan at 4.75%,

Exact monthly payment = $(350{,}000 \times .0475/12)/[1 - (1 + .0475/12)^{-360}]$ = $1,825.77

Approximate monthly payment = $(179 + 71 \times 4.75) \times 3.5$ = $1,806.88

The difference is $18.89 which is only 1.03% error. If the mortgage rate increases to 7.0%,

Exact monthly payment = $(350{,}000 \times .07/12)/[1 - (1 + .07/12)^{-360}]$ = $2,328.56

Approximate monthly payment = (179 + 71 x 7) x 3.5 = \$2,366

The difference is \$37.44 which is only 1.6% error.

2.3. 646+65-14 (Combined) Rule

Now, the 646+65-14 combined rule is proposed for both 15- and 30-year mortgage terms.

Approximate monthly payment = (646 + 65 x i – 14 x N) x P (3)

where N is either 15 or 30. Since this rule covers both 15- and 30-year terms, it is more convenient to use this rule than two independent 15- and 30-year approximate rules proposed earlier. Generally, however, this combined rule tends to show slightly higher percentage errors than those two independent rules depending on mortgage interest rates. Consider a 15-year, \$300,000 mortgage loan at 4.75%.

Exact monthly payment = (300,000 x .0475/12)/[1 – $(1 + .0475/12)^{-180}$] = \$2,333.50

Approximate monthly payment = (646 + 65 x 4.75 – 14 x 15) x 3 = \$2,234.25

The difference is \$99.25 which is 4.25% error. If the mortgage rate increases to 6.0%,

Exact monthly payment = (300,000 x .06/12)/[1 – $(1 + .06/12)^{-180}$] = \$2,531.57

Approximate monthly payment = (646 + 65 x 6 – 14 x 15) x 3 = \$2,478

The difference is \$53.57 which is 2.12% error. If this is a 30-year mortgage loan,

Exact monthly payment = (300,000 x .06/12)/[1 – $(1 + .06/12)^{-360}$] = \$1,798.65

Approximate monthly payment = (646 + 65 x 6 – 14 x 30) x 3 = \$1,848

The difference is \$49.35 which is 2.74% error. If the mortgage rate decreases to 4.25%,

Exact monthly payment = (300,000 x .0425/12)/[1 – $(1 + .0425/12)^{-360}$] = \$1,475.82

Approximate monthly payment = (646 + 65 x 4.25 – 14 x 30) x 3 = \$1,506.75

The difference is \$30.93 which is 2.10% error.

As a matter of fact, even if the combined rule has slightly higher percentage errors, it may be preferred as one single rule that covers both 15- and 30-year mortgage terms. Once again, this combined rule also can be used for any amount of the mortgage loan.

3. DEVELOPMENT OF APPROXIMATE RULES

According to Mortgage-X.com (http://mortgage-x.com/trends.htm), the national average contract mortgage rate ranges from 4% to 15% for the past 5 decades. Approximate rules are developed within this range. As we know, basically, there are two different types of annuity. One is the ordinary annuity and the other is the annuity-due. The monthly payment formula for each annuity is as follows.

Ordinary Annuity

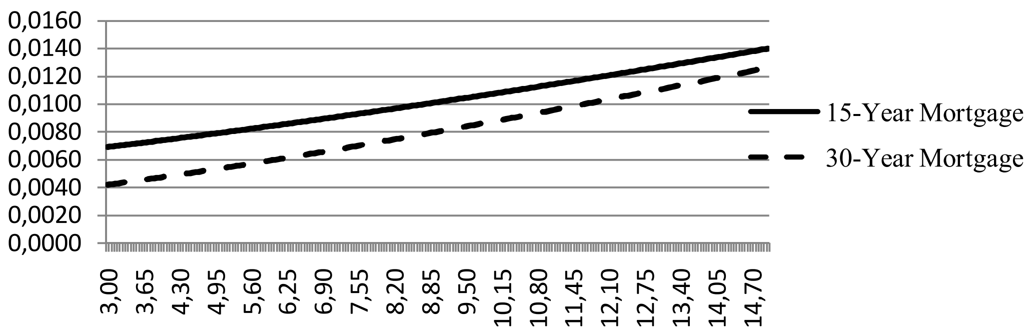
$R = (i/12)/[1 - (1 + i/12)^{-n}] \times P$ (4)

Annuity-Due

$$R = (i/12)/[(1 + i/12) - (1 + i/12)^{-n+1}] \times P \quad (5)$$

where i is an interest rate, R is the monthly payment at i, n is the number of months, and P is a loan amount. Since, however, the actual mortgage payment is calculated on the basis of the ordinary annuity, our approximate rules are developed using Equation (4). Under the assumption that the loan amount is $1, exact monthly payments are calculated as the interest rate increases from 3% to 15% given an increment of 0.05%. Figure 1 shows how monthly payments increase for both 15-year and 30-year mortgage terms as the interest rate increases.

Figure 1: Monthly Payments



Both curves are flat, and the 30-year curve is slightly steeper than the 15-year curve. R is an increasing function of i, and its curvatures for i values (4% - 15%) are small, which also confirms that R is a flat curve. This makes it possible to construct an approximate linear relationship between R and i. The following simple linear regression equation is used.

$$R(i) = \alpha + \beta i + \gamma D + \varepsilon \quad (6)$$

where D is a dummy variable that has either 15 or 30 depending on the mortgage term. First, two independent 15- and 30-year approximate rules are developed without the dummy variable. Second, the combined rule is developed with the dummy variable set to 15 if the mortgage term is 15 years and 30 if the mortgage term is 30 years. The regression results are shown in Table 1. Adjusted R-squares for all three cases are close to one, and coefficients are very significant at 1% level. As a result, we obtain a well-fitted linear relationship between monthly payments and interest rates.

Table 1: Regression Results

We calculate monthly payments at interest rates from 3% to 15% using an increment of .05%. Then, we regress monthly payments computed on interest rates and a dummy variable.

*** indicates a statistical significance at 1%.

Mortgage Term	σ	β	γ (Dummy)	Adjusted R^2
15-Year Term	.00492 (t=295.14***)	.00059 (t=343.27***)	-	.9980
30-Year Term	.00179 (t=85.97***)	.00071 (t=328.01***)	-	.9978
Combined Term	.00646 (t=151.52***)	.00065 (t=216.85***)	-.00014 (t=-99.26***)	.9916

Using coefficients, σ and β estimated from three regressions in Table 1, we write three approximate rules as follows.

15-year mortgage Rule

Approximate Monthly Payment = (492 + 59 x *i*) x P

30-year mortgage Rule

Approximate Monthly Payment = (179 + 71 x *i*) x P

Combined Rule

Approximate Monthly Payment = (646 + 65 x *i* – 14 x N) x P

where *i* is an interest rate (%), N is either 15 or 30, and P is a loan amount per $100,000.

In Table 2, the percentage error is calculated as the absolute value of the ratio of the difference between the exact payment and the approximate payment to the exact payment.

With the historical range of the national average contract mortgage rate (4% - 15%), most cases have less than 3% errors. Although the combined rule shows slightly higher percentage errors than two independent 15- and 30-year rules, it may be more convenient because it covers both 15- and 30-year terms with one single equation. It, however, entirely depends on users' preference. Since regressions are based on $1 loan, all three rules work in exactly the same way for any amount of the mortgage loan.

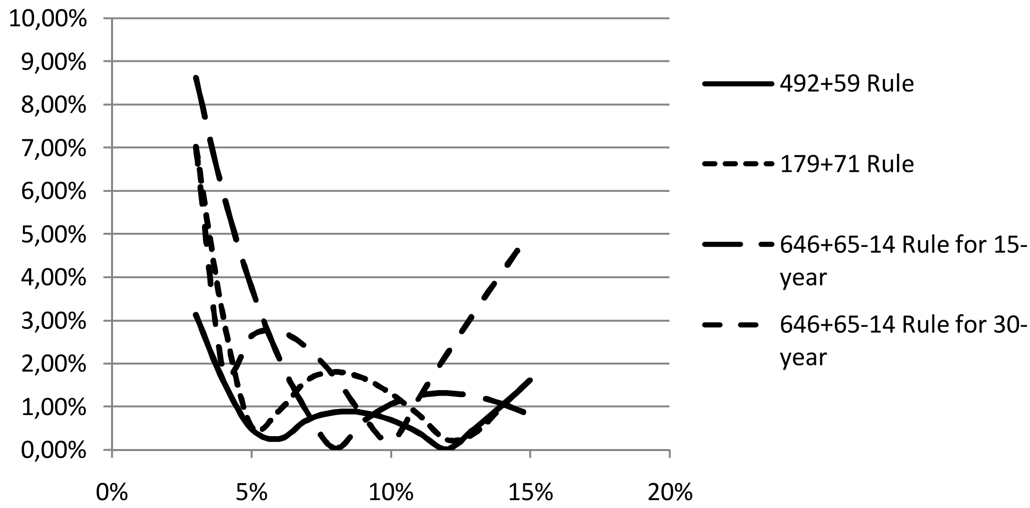
Table 2: Percentage Errors of Approximate Payments

% Error is computed as the absolute value of the ratio of the difference between the exact payment and the approximate payment to the exact payment.

Interest Rate	% Error for 15-Year Mortgage Term (492+59 rule)	% Error for 30-Year Mortgage Term (179+71 rule)	% Error for Combined Term (646+65-14 rule)	
			15-Year Term	30-Year Term
3%	3.13%	7.02%	8.63%	7.02%
4%	1.58%	3.02%	5.91%	1.80%
5%	0.48%	0.53%	3.77%	2.64%
6%	0.25%	0.91%	2.12%	2.74%
7%	0.69%	1.61%	0.87%	2.36%
8%	0.87%	1.80%	0.04%	1.67%
9%	0.86%	1.66%	0.66%	0.79%
10%	0.69%	1.30%	1.06%	0.18%
11%	0.39%	0.81%	1.27%	1.19%
12%	0.01%	0.23%	1.32%	2.20%
13%	0.49%	0.38%	1.25%	3.18%
14%	1.03%	1.00%	1.07%	4.12%
15%	1.61%	1.62%	0.82%	5.02%

Figure 2 also shows percentage errors graphically for all three rules. It is certain that percentage errors for all rules are relatively small (less than 2%) in the range between 5% and 10% which was dominant during the past two decades.

Figure 2: Percentage Errors for Approximate Rules



4. CONCLUSION

It is true that there are many tools that can be used to calculate the monthly mortgage payment. Sometimes, however, they may not be immediately available. Since there are few approximate rules known to people, we propose three approximate rules under the assumption that any specific tools are not immediately available. The 492+59 rule is an approximate method to calculate a 15-year mortgage payment and the 179+71 rule is an approximate method to calculate a 30-year mortgage payment. These approximate rules are very convenient and show quite small errors over the historical range of the national average contract mortgage rate. The combined 646+65-14 rule is even more convenient to use but shows a little higher errors than two independent rules.

Not only financial professionals but also academicians can use either two independent 15- and 30-year rules or the combined rule. These approximate rules can be widely used in financial education, college classroom teaching, and even financial consulting work. Since these rules show trivial errors that can be ignored in any informal situation, it is expected that these rules help make a quick mortgage decision as one of important investment decisions.

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